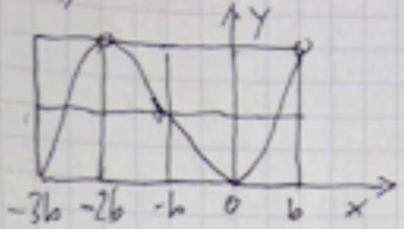
Affenkasten - Beweise



$f(x) = a(x^2(x+3b))$

$a=1$

$f(x) = x^3 + 3bx^2$ ↑ in der Schnelle vergessen ☹

$f'(x) = 3x^2 + 6bx$

$f'(x) = 0 \Leftrightarrow 3x(x+2b) = 0$

$\Leftrightarrow x = 0 \vee x = -2b$

zu zeigen ist $f(-2b) = f(b)$

$f(-2b) = (-2b)^3 + 3b(-2b)^2$

$= -8b^3 + 12b^3 = 4b^3$

$f(b) = b^3 + 3bb^2 = 4b^3$] (=)

Übrigens WP:

$f''(x) = 6x + 6b$

$f''(x) = 0 \Leftrightarrow 6x + 6b = 0$

$\Leftrightarrow \underline{x = -b}$ wie erwartet

$f(-b) = (-b)^3 + 3b(-b)^2$

$= -b^3 + 3b^3 = 2b^3 = \frac{1}{2}(4b^3)$

↑ wie erwartet

Mit $a \neq 1$ ist die Zellenhöhe $\boxed{2ab^3}$ Kastenhöhe $\boxed{4ab^3}$

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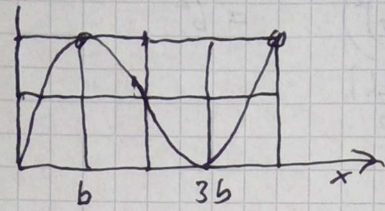

$g(x) = a(x(x-3b)^2)$

$a=1$

$g(x) = x(x^2 - 6bx + 9b^2)$

$g(x) = x^3 - 6bx^2 + 9b^2x$

$g'(x) = 3x^2 - 12bx + 9b^2$

$= 3(x^2 - 4bx + 3b^2)$

QGl $x^2 - 4bx + 3b^2 = 0$

$x^2 - 4bx + (2b)^2 = 4b^2 - 3b^2$

$(x-2b)^2 = b^2$

$x - 2b = \pm b$

$x = 3b$ ✓ $x = b$

war klar. linke Max-stelle

zu zeigen: $\boxed{g(b) = g(4b)}$

$g(b) = b^3 - 6bb^2 + 9b^2b = 4b^3$

$g(4b) = (4b)^3 - 6b(4b)^2 + 9b^2(4b)$

$= 64b^3 - 6b \cdot 16b^2 + 36b^3$

$= 64b^3 - 96b^3 + 36b^3 = 4b^3$ (=)

WP: $g''(x) = 3(2x - 4b)$

$g''(x) = 0 \Leftrightarrow 2x - 4b = 0 \Leftrightarrow x = 2b$ wie erwartet

$g(2b) = 2b(4b^2 - 12b^2 + 9b^2)$

$=$