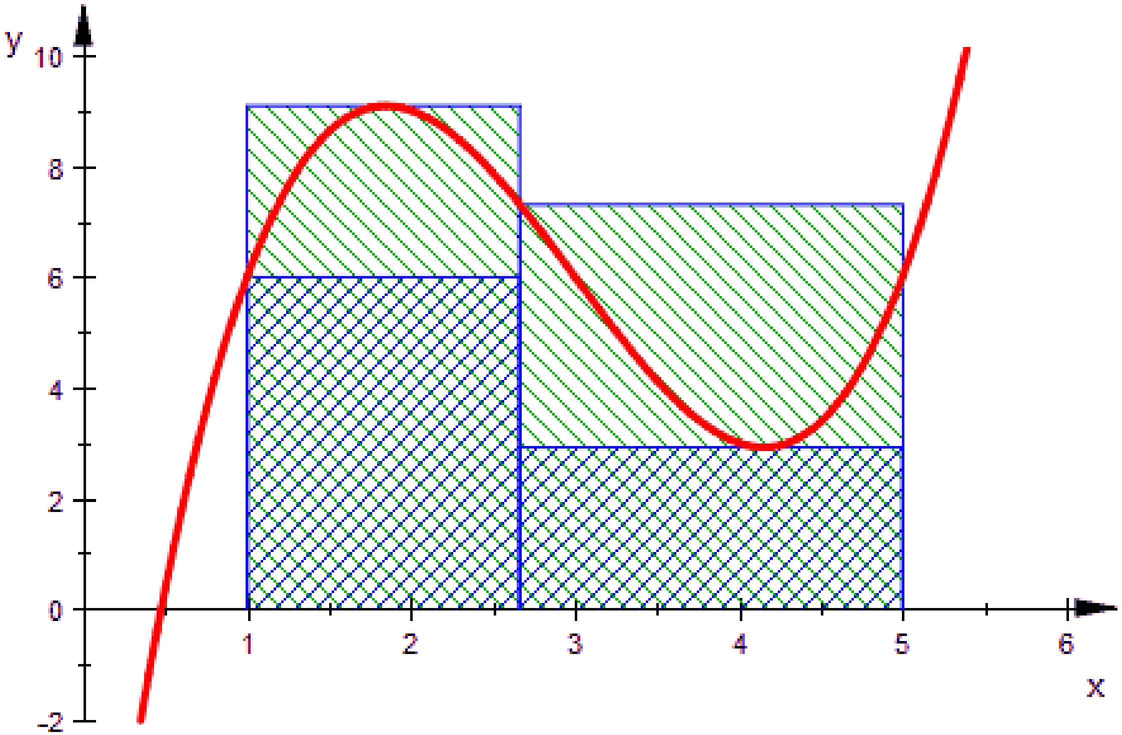
Infinitesimales

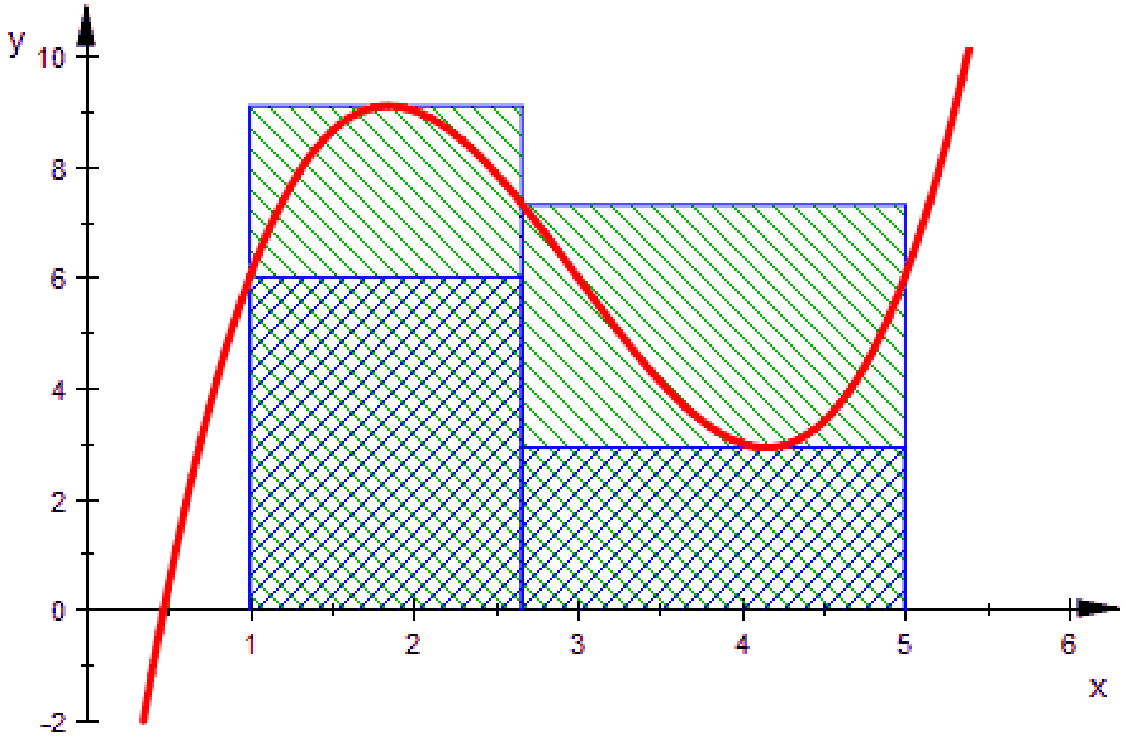
Hier wächst Ihr Wissen über das unendlich Kleine



Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2103 http://www.leuphana.de/matheomnibus

Infinitesimal Thinking

Your knowledge about infinitely small objects increases.



Der Modellierungskreislauf

Ein erfundenes Beispiel:

16 Uhr Unfall mit Fahrerflucht in Hann. Münden

Ein Zeuge glaubt einen Transporter mit reichlich Werbeschrift gesehen zu haben.

Der Besitzer behauptet er sei um 16 Uhr gar nicht in Hann.Münden gewesen.

The Modelling Circuit

A faked example:

At 4 pm o‘clock there had been an accident in H.-Münden, the driver escaped.

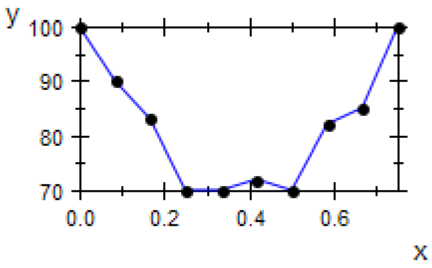
A witness had seen a van with multiple commertial marking how the picture shows.

The owner affirm that at 4 o‘clock he had not been in H.-Münden.

Der Modellierungskreislauf

Um 15 Uhr war er nachweislich noch in Bodenwerder, 80 km entfernt.

Der Fahrtenschreiber zeigt:



Reale Situation

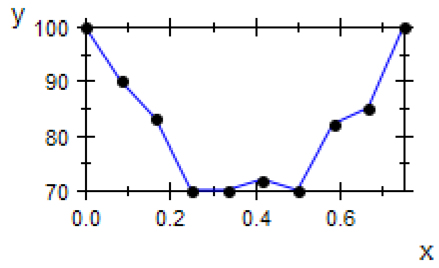
Länge der gefahrenen Strecke ist gesucht.

•Die Weser entsteht in Hannoversch Münden durch Zusammenfluss von Werra und Fulda.
Sie durchfließt Niedersachen bis zur Nordsee. In Bodenwerder ist das Schloss des Lügenbarons Freiherr von Münchhausern. Er zog sich am eigenen Zopf aus dem Sumpf usw….

The Modeling Circuit

At 5 pm o‘clock he was verifiably still in Bodenwerder*, 80 km downstream the river Weser.

The trip recorder shows:



real situation

We are interested in the length of his drive.

*Notice: the river Weser starts in Hannoversch Münden in Lower Saxony and goes in the North Sea. In Boderwerder is the castle of the „Lying Lord Münchhausen“.

Der Modellierungskreislauf

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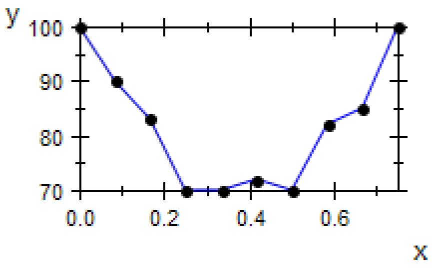

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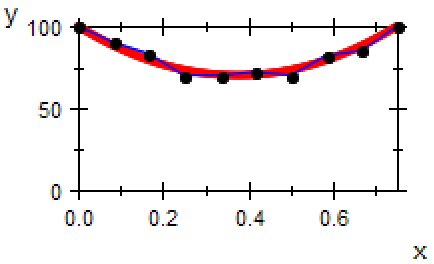

mathematisches Modell

Fläche unter der Modellkurve gesucht.

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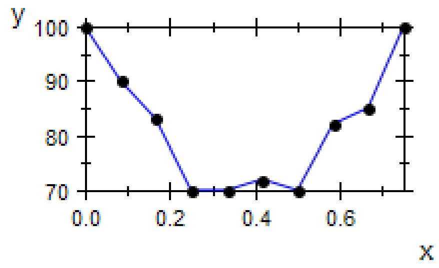

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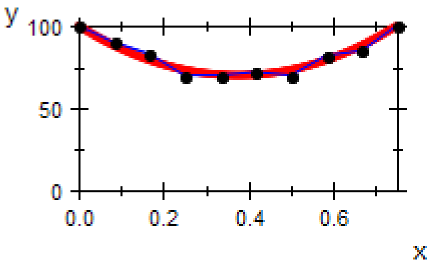

mathematical model

We search the area under the modeling curve.

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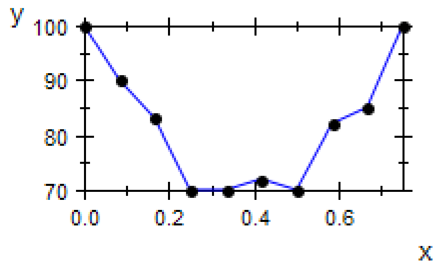

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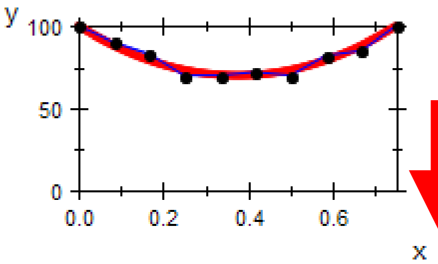

mathematisches Modell

Fläche unter der Modellkurve gesucht.

mathematische Lösungsidee

$$s = \int_0^{0.75} v(t)\, dt$$

The Modeling Circuit

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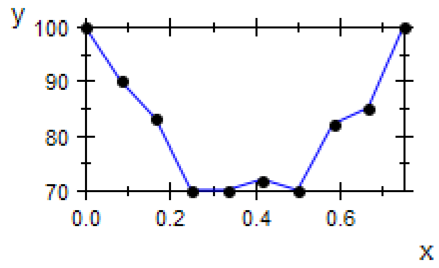
The trip recorder shows:



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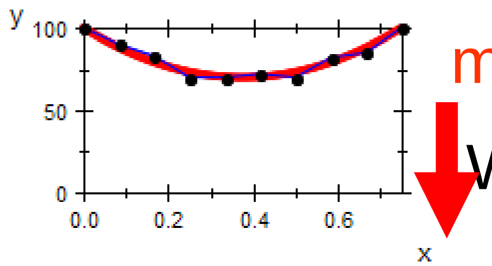

mathematical model

We search the area under the modeling curve.

mathematical idea of solving this

$$s = \int_{0}^{0.75} v(t)\, d\, t$$

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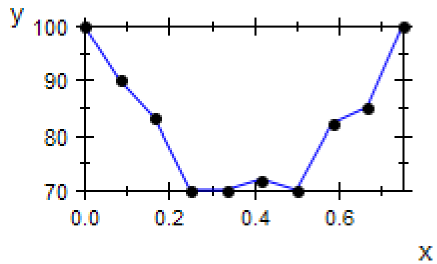

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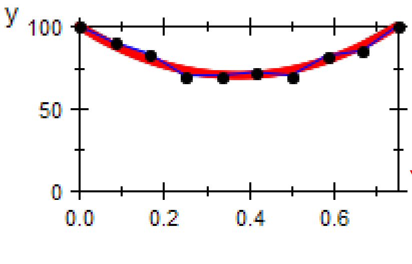

mathematisches Modell

Fläche unter der Modellkurve gesucht.

mathematische Lösungsidee

$$s = \int_{0}^{0.75} v(t)\, d\, t$$

mathematische Antwort

$$s = 60\ km$$

The Modeling Circuit

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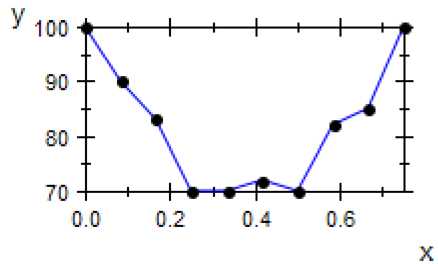

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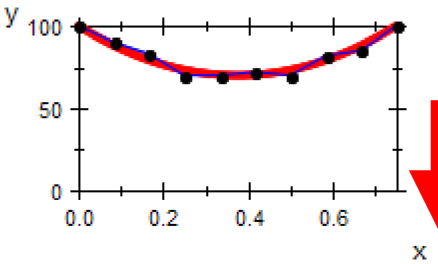


mathematical model

We search the area under the modeling curve.

mathematical idea of solving this

$$s = \int_{0}^{0.75} v(t)\, d\,t$$

mathematical solution

$$s = 60\ km$$

Funktionen werden zum Werkzeug

Man erhält Antworten beim

Blick auf „das Ganze“ mit dem Integral

integer (lat.)= ganz

pane integrale (it.) = Vollkornbot

$$\int f(x)dx$$

Funktionen beschreiben Zusammenhänge

Man erhält punktuelle Antworten mit dem Differential

$$df, \quad \frac{dy}{dx}, \quad f'(x)$$

Functions Become a Tool

You have solution with looking
on on the whole issue with the Integral

integer (lat.)= whole

pane integrale (it.) = whohe –grain breadn

$$\int f(x)dx$$

functions are describing connections

Youe have punctual solutions
with the differential

$$df, \quad \frac{dy}{dx}, \quad f'(x)$$

Das Integral

$$\int f(x)dx$$

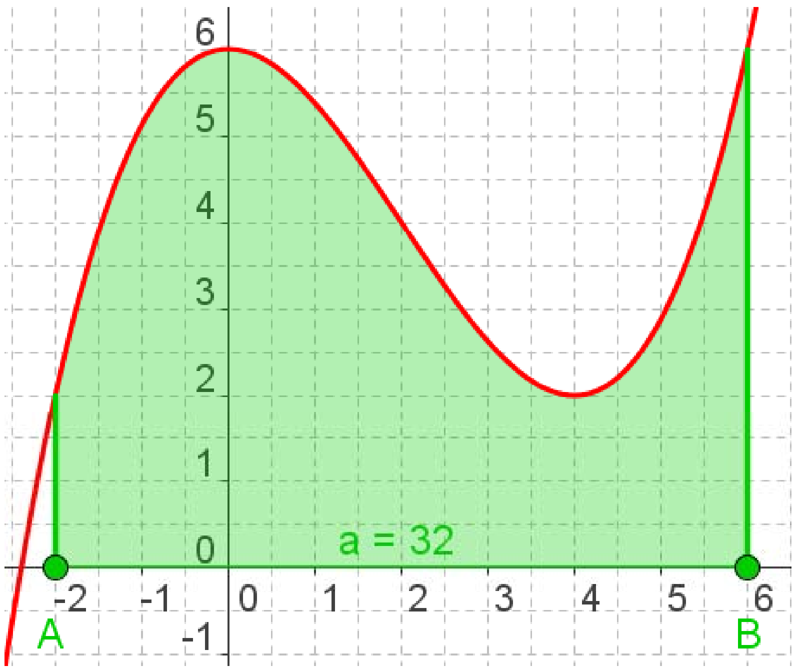
Man erhält Antworten beim

Blick auf „das Ganze“ mit dem Integral

$$\int_a^b f(x)dx$$

integer (lat.)= ganz

pane integrale (it.) = Vollkornbot



The Integral

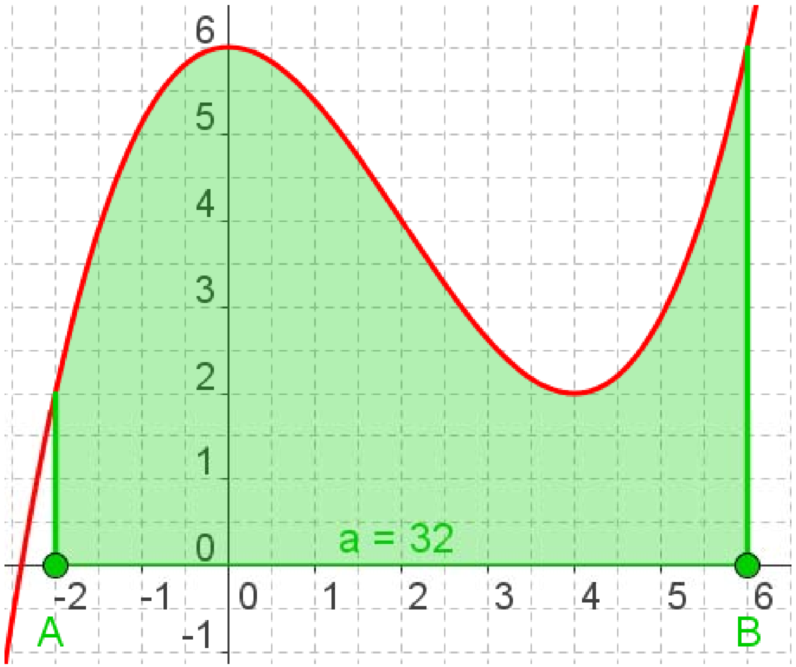
$$\int f(x)dx$$

You have solution with looking
on on the whole issue with the Integral

integer (lat.)= whole

$$\int_a^b f(x)dx$$

pane integrale (it.) = whohe –grain bread



Das Riemannsche Integral

$$\int_a^b f(x)d\,x$$

Bernhard Riemann

Abi 1846

Johanneum Lüneburg

Originaltext aus „Gesammelte Werke"

Ueber den Begriff eines bestimmten Integrals und den Umfang seiner Gültigkeit.

4.

Die Unbestimmtheit, welche noch in einigen Fundamentalpunkten der Lehre von den bestimmten Integralen herrscht, nöthigt uns, Einiges voraufzuschicken über den Begriff eines bestimmten Integrals und den Umfang seiner Gültigkeit.

Also zuerst: Was hat man unter $\int_a^b f(x)\,dx$ zu verstehen?

The Riemannian Integral

$$\int_a^b f(x)dx$$

Bernhard Riemann

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Ueber den Begriff eines bestimmten Integrals und den Umfang seiner Gültigkeit.

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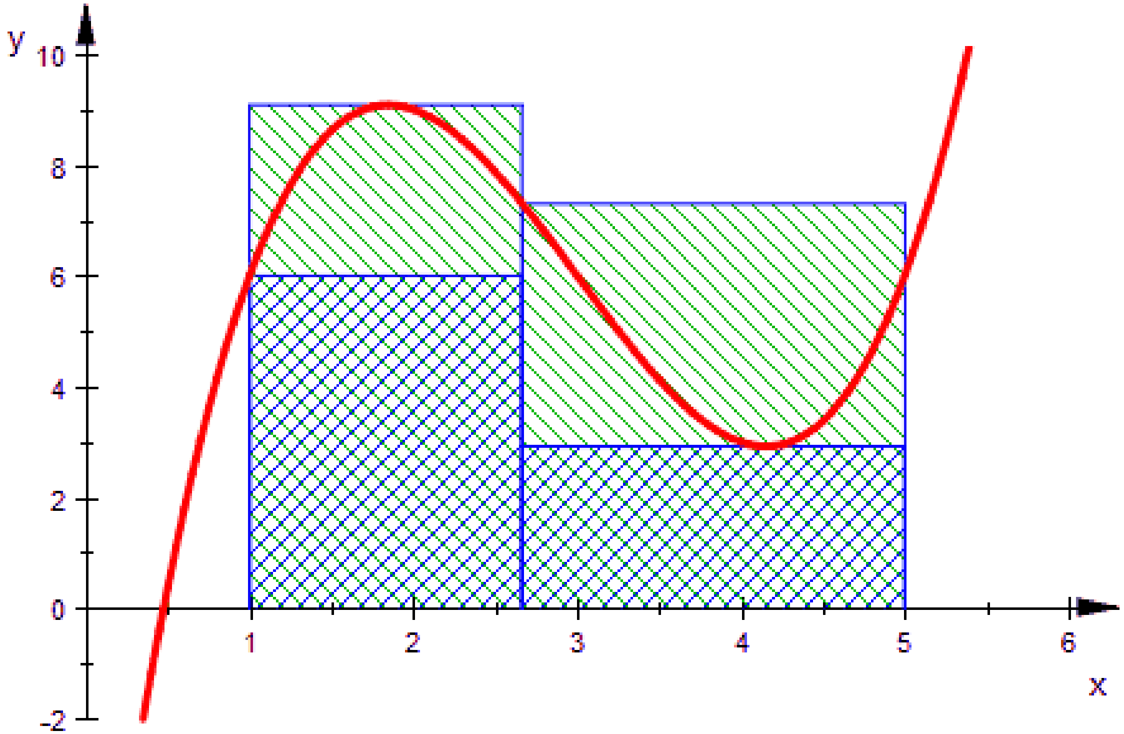
Also zuerst: Was hat man unter $\int_a^b f(x)\,dx$ zu verstehen?

On the conception of the definite integral and the range of its validity.

……

So at first: What is the meaning of $\int_a^b f(x)dx$?

$\int_a^b f(x)dx$ Riemannsches Integral



Bernhard Riemann

Abi 1846

Johanneum Lüneburg

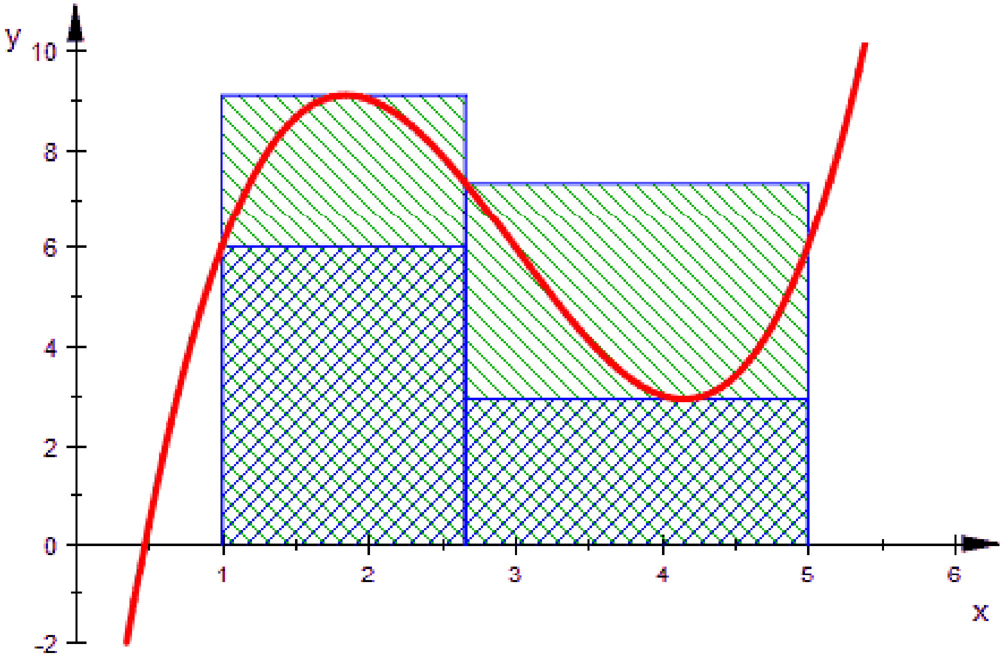
Hat sie diese Eigenschaft nicht, so hat $\int_a^b f(x)\,dx$ keine Bedeutung.

die Fkt f

, bei jeder Zerlegung denselben Grenzwert zu haben,

Originaltext aus „Gesammelte Werke“

$\int_a^b f(x)dx$ Riemannian Integral



Bernhard Riemann

Abitur 1846

Johanneum Lüneburg

Hat sie diese Eigenschaft nicht, so hat $\int_a^b f(x)\,dx$ keine Bedeutung.

If the function has not this property, so the symbol $\int_a^b f(x)dx$ has no meaning.

, to have the same limit with every dissection,

original text out of „Gesammelte Werke“

Das Integral als verallgemeinertes Produkt

v konstant $\quad s = v \cdot t \quad$ $v = v(t)$ variabel

Geschwindigkeit Weg Zeit

$$s = \int_a^b v(t)\, d\, t$$

Integral für 3D-Flächen, Volumen, Schwerpunkt, Bilanzen....

The Integral as a Generalized Product

v constant — $s = v \cdot t$ — $v = v(t)$ variable

velocity — path — time

$$s = \int_a^b v(t)\, dt$$

Integral for 3D-areas, volumes, balance points, balances,…

Das Integral als verallgemeinertes Produkt

v konstant $\quad s = v \cdot t \quad$ $v = v(t)$ variabel

Geschwindigkeit Weg Zeit

$$s = \int_a^b v(t)\,dt$$

F konstant $\quad W = F \cdot s$

Kraft Arbeit Weg

$$F = F(s)$$

$$W = \int_a^b F(s)\,ds$$

Integral für 3D-Flächen, Volumen, Schwerpunkt, Bilanzen....

The Integral as a Generalized Product

v constant | $s = v \cdot t$ | $v = v(t)$ variable

velocity | path time

$$s = \int_a^b v(t)\, dt$$

F constant | $W = F \cdot s$ | $F = F(s)$

forth | work energy

$$W = \int_a^b F(s)\, ds$$

Integral for 3D-areas, volumes, balance points, balances,…

Das Integral als verallgemeinertes Produkt

v konstant $\qquad s = v \cdot t \qquad v = v(t)$ variabel

Geschwindigkeit Weg Zeit

$$s = \int_a^b v(t)\, d\,t$$

F konstant $\qquad W = F \cdot s \qquad F = F(s)$

Kraft Arbeit Energie Weg

$$W = \int_a^b F(s)\, d\,s$$

R konstant $\qquad U = R \cdot I \qquad R = R(I)$ variabel

Widerstand Spannung Stromstärke

$$U = \int_a^b R(I)\, d\,I$$

Integral für 3D-Flächen, Volumen, Schwerpunkt, Bilanzen....

The Integral as a Generalized Product

v constant $\quad s = v \cdot t \quad v = v(t)$ variable

velocity $\quad$ path $\quad$ time

$$s = \int_a^b v(t)\, d\,t$$

F constant $\quad W = F \cdot s \quad F = F(s)$

forth $\quad$ work energy

$$W = \int_a^b F(s)\, d\,s$$

R constant $\quad U = R \cdot I \quad R = R(I)$ variable

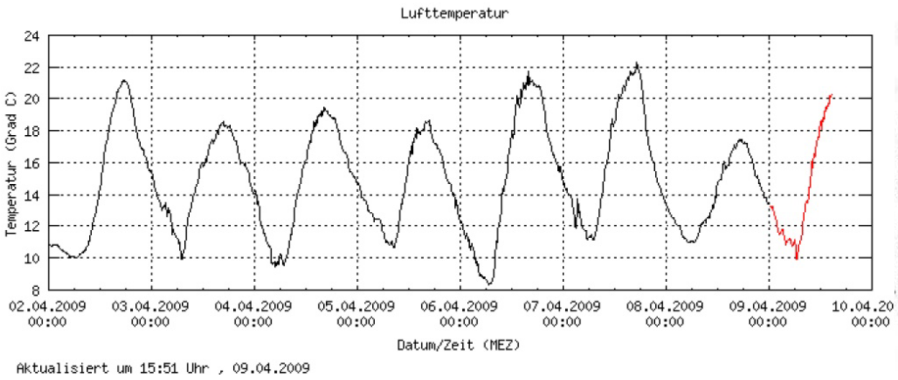
resistor $\quad$ voltage $\quad$ electric current

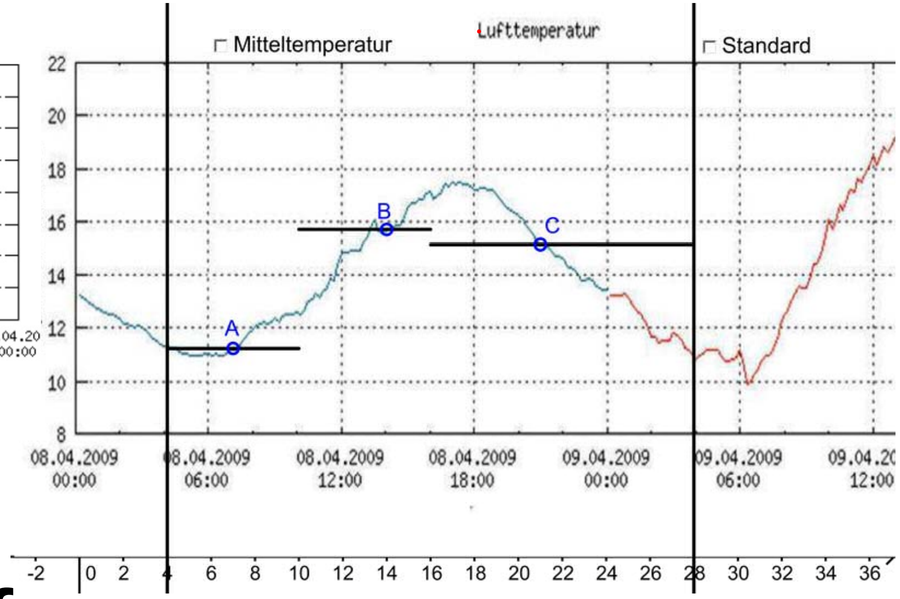
Ohn‘s law

$$U = \int_a^b R(I)\, d\,I$$

Integral for 3D-areas, volumes, balance points, balances,…

Das Integral für den verallgemeinerten Mittelwert



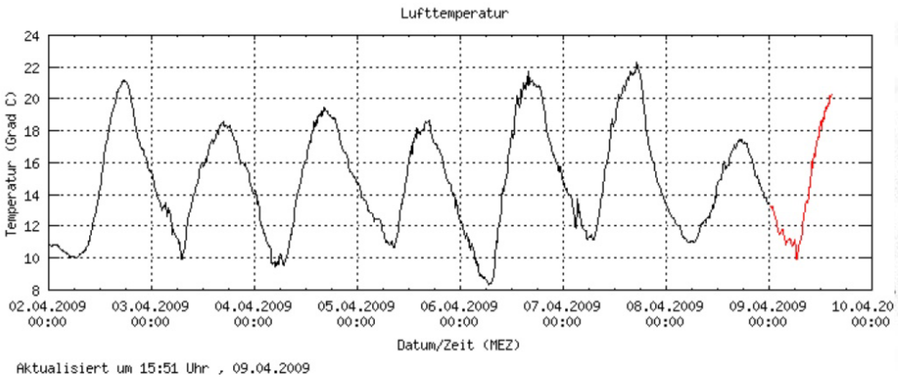


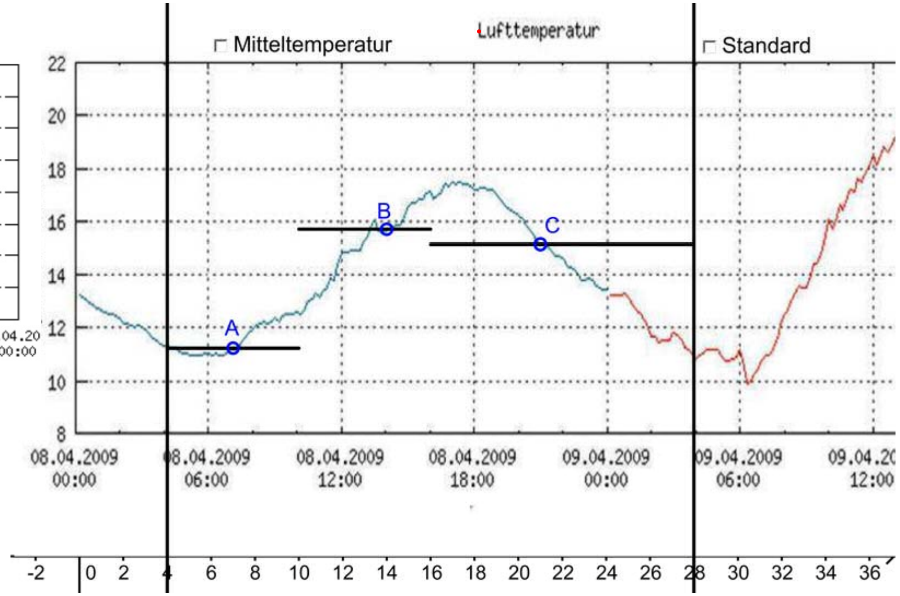
Wetter
Temperaturverlauf

$$T_{\mathbf{mittel}} = \frac{1}{4}\left(T_7 + T_{14} + 2 \cdot T_{21}\right)$$

Integral für Mittelwert und Bilanzen....

The Integral for the Generalized Mean





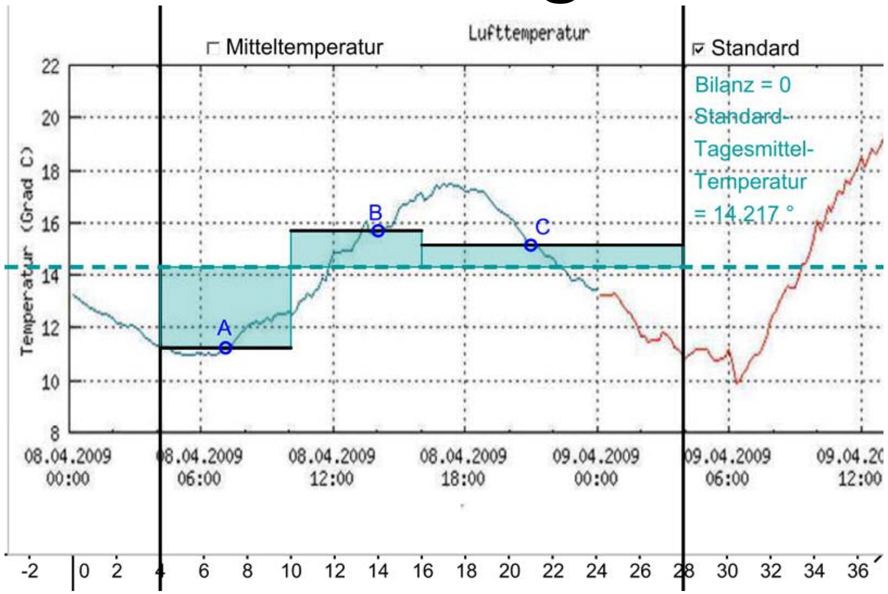
weather

temperature profile

$$T_{\mathbf{mean}} = \frac{1}{4}\left(T_7 + T_{14} + 2 \cdot T_{21}\right)$$

Integral for means and financial balances....

Das Integral für den verallgemeinerten Mittelwert



Ist die Modellierung der

Metereologen

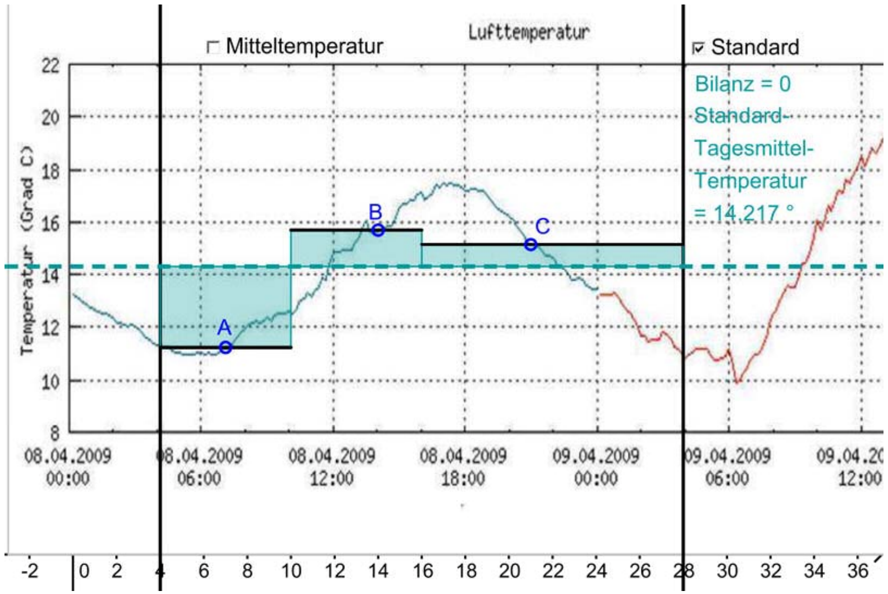
nicht viel zu grob?????

$$T_{\mathbf{mittel}} = \frac{1}{4}\left(T_7 + T_{14} + 2 \cdot T_{21}\right)$$

Flächenbilanz=0

Integral für Mittelwert und Bilanzen....

The Integral for the Generalized Mean



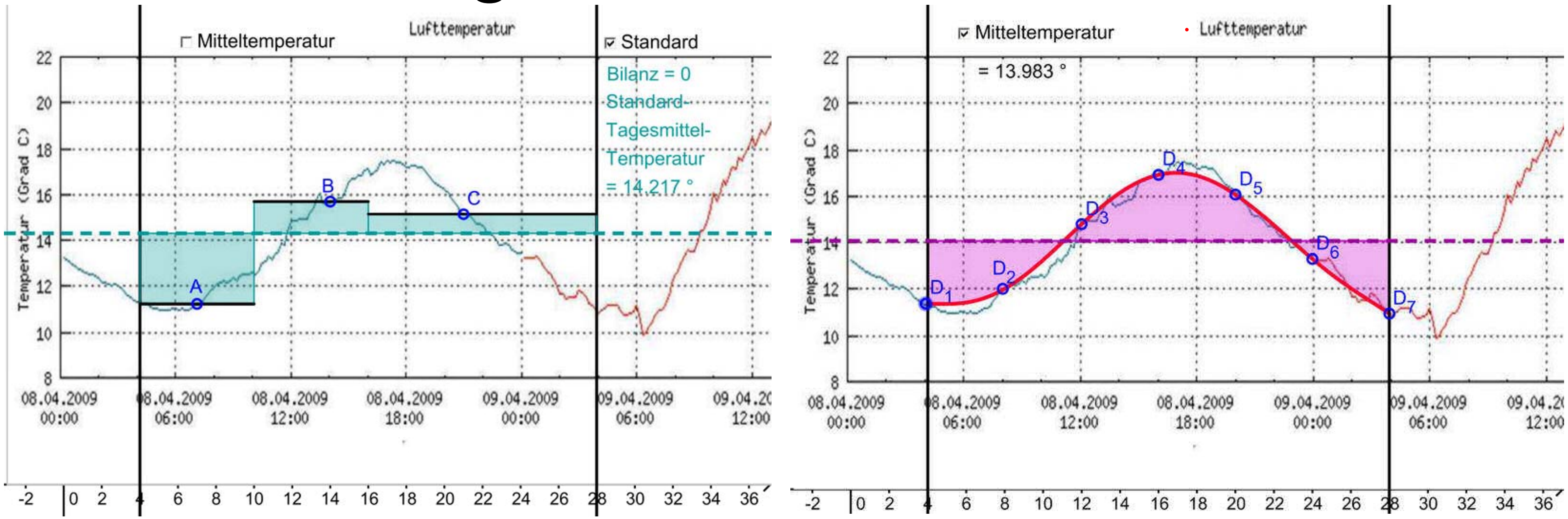
Is the modeling of the meterologists too rough?

$$T_{\mathbf{mean}} = \frac{1}{4}\left(T_7 + T_{14} + 2 \cdot T_{21}\right)$$

balance of area =0

integral for means and balances....

Das Integral für den verallgemeinerten Mittelwert



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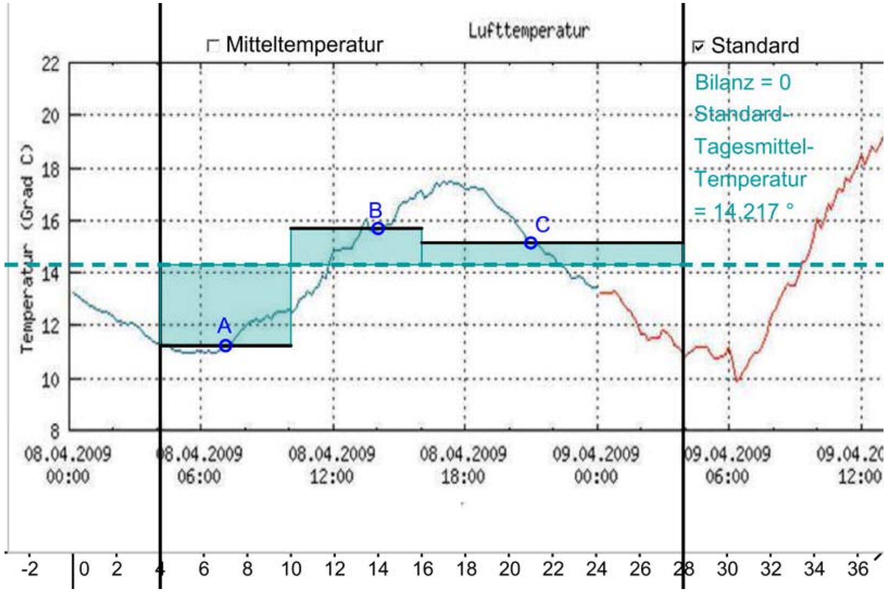
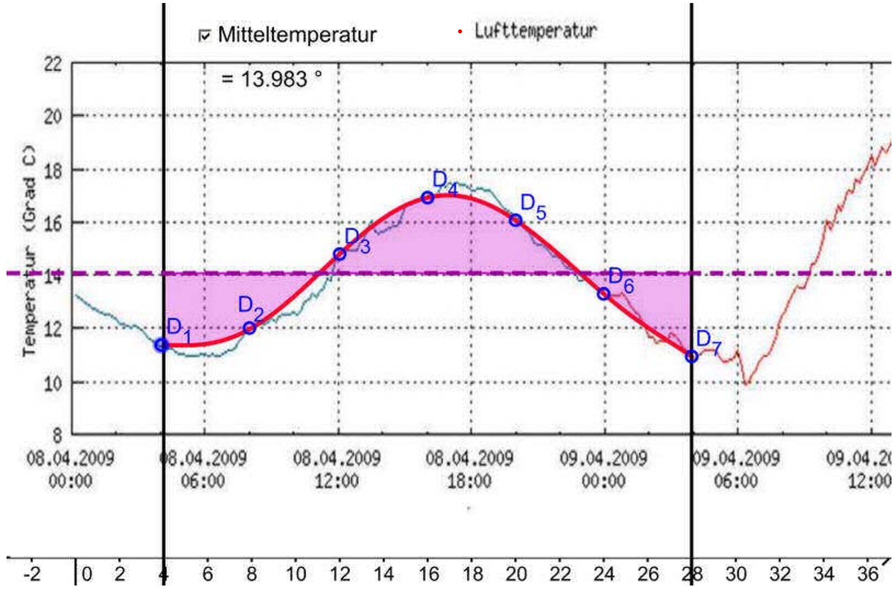
Integral für Mittelwert und Bilanzen....

The Integral for the Generalized Mean





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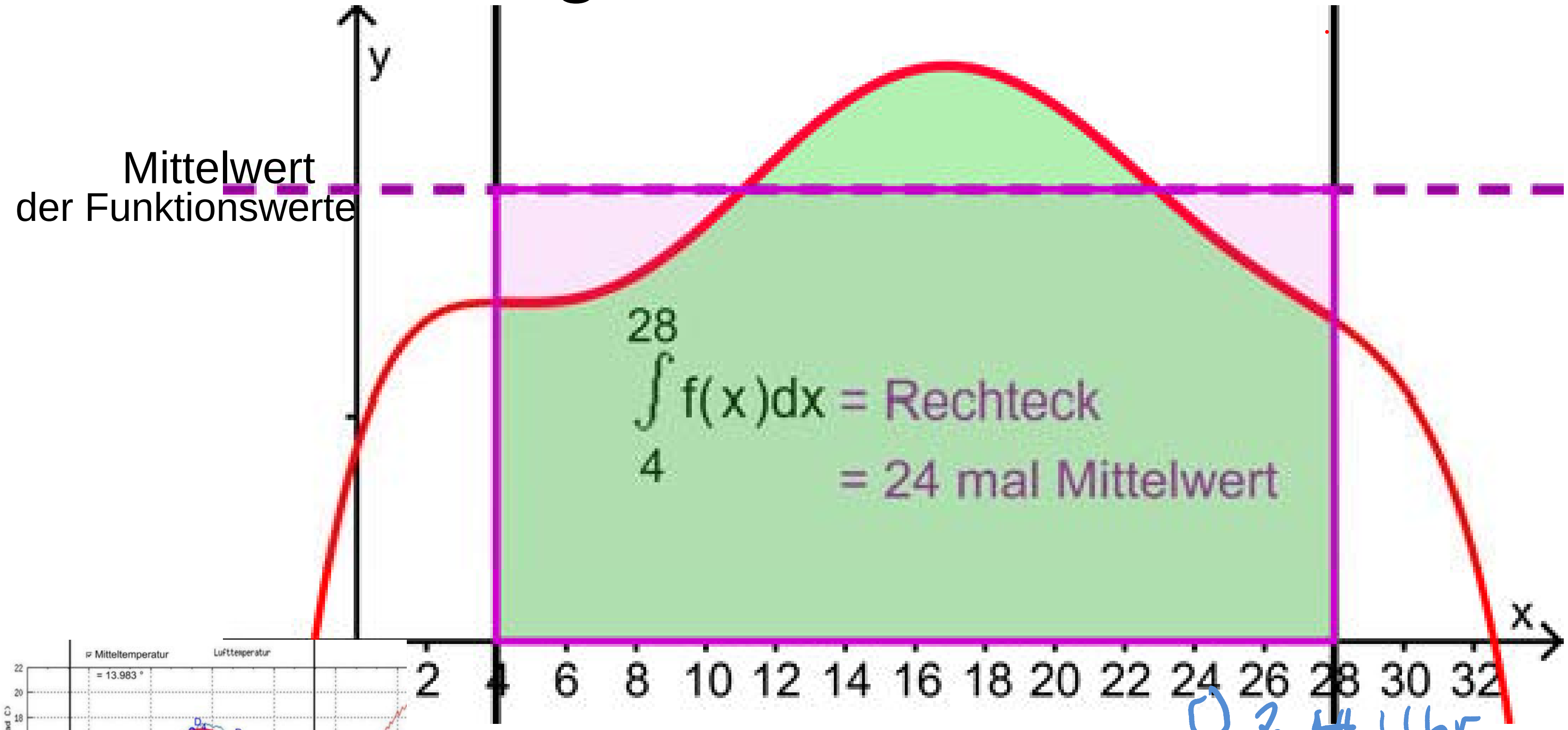
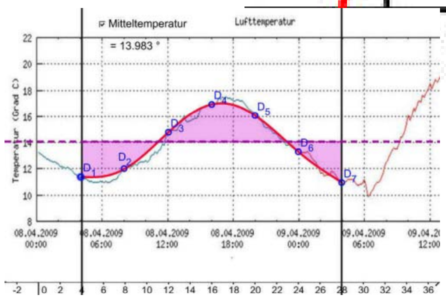
integral for means and balances....

Das Integral für den verallgemeinerten Mittelwert

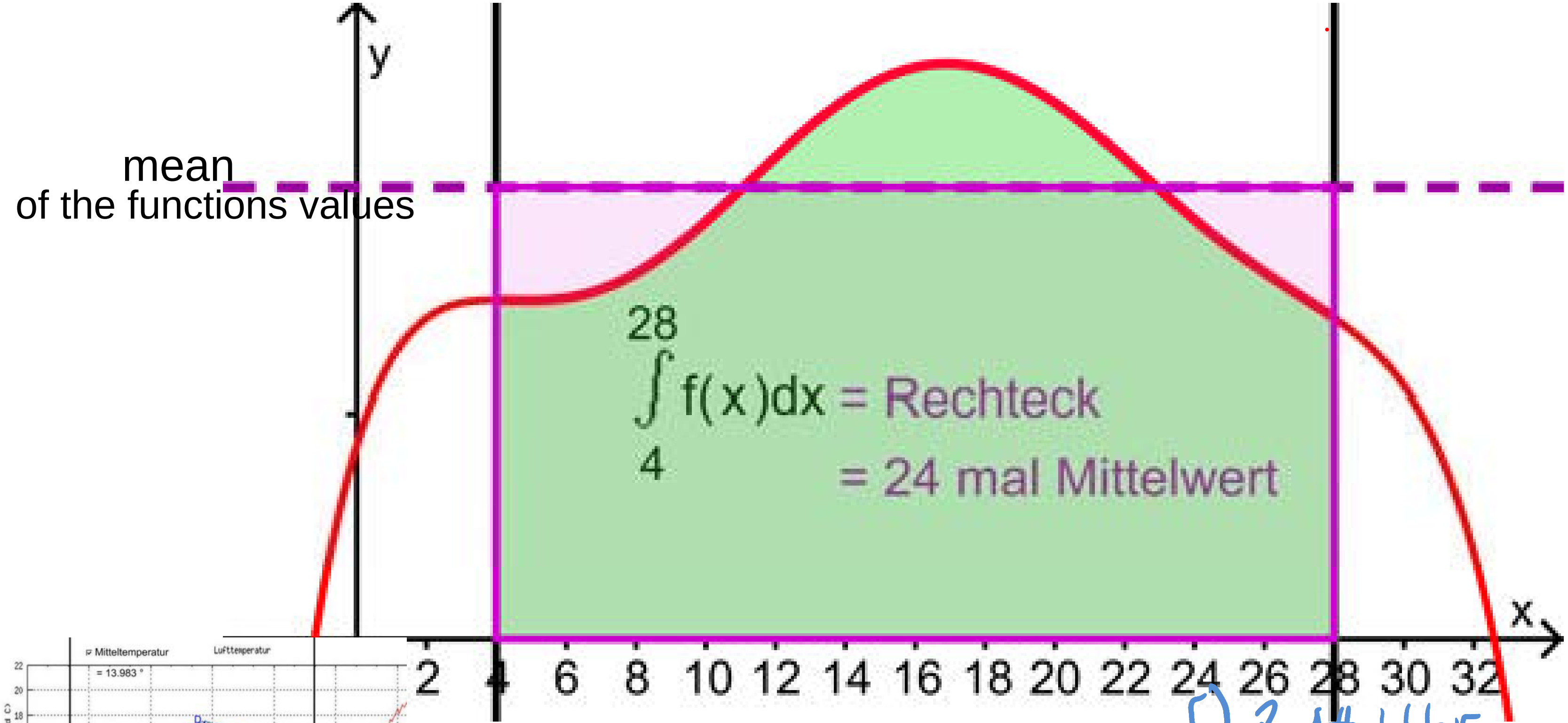


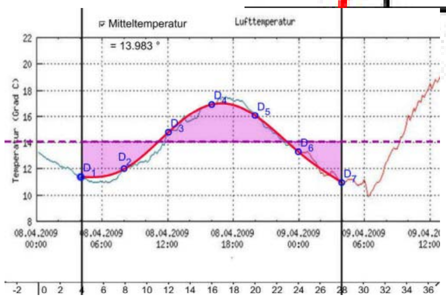


$$\text{Mittelwert} = \frac{1}{b-a}\int_a^b f(x)dx$$

Integral für 3D-Flächen, Volumen, Schwerpunkt, Bilanzen....

The Integral for the Generalized Mean

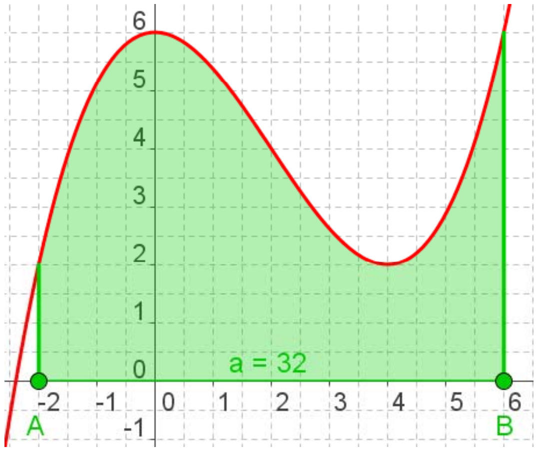




$$\boxed{\text{mean} = \frac{1}{b-a}\int_a^b f(x)dx}$$

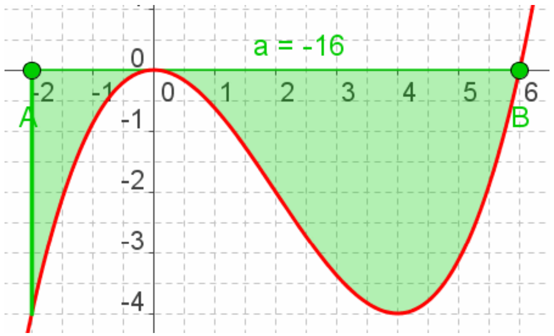
integral 3D-areas aund volumes, for means and balances..

Eigenschaften des Integrals



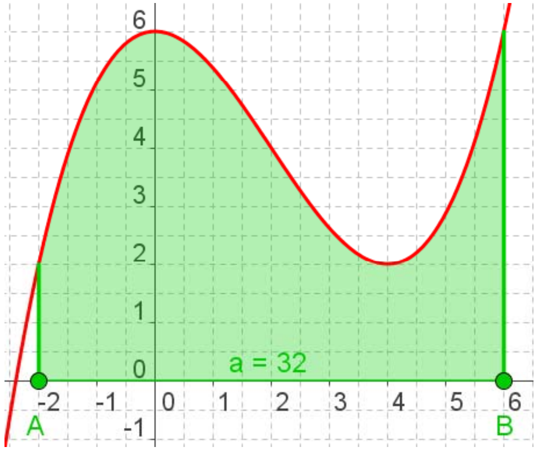
$$\int_a^b f(x)\,dx$$

Intervall [A,B]



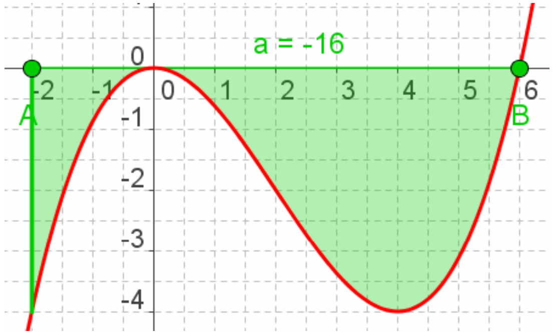
Sind die Werte von f im ganzen Intervall negativ, dann ist auch das Integral negativ.

Properties of the Integrals



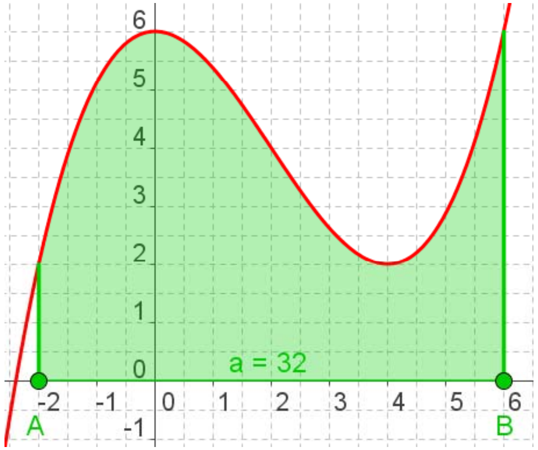
$$\int_a^b f(x)dx$$

interval [A,B]



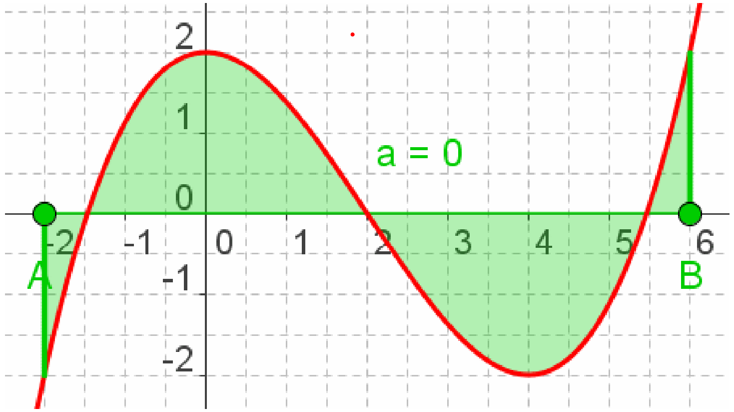
If the values of f are negative in the whole interval
than the integral is negativ.

Eigenschaften des Integrals

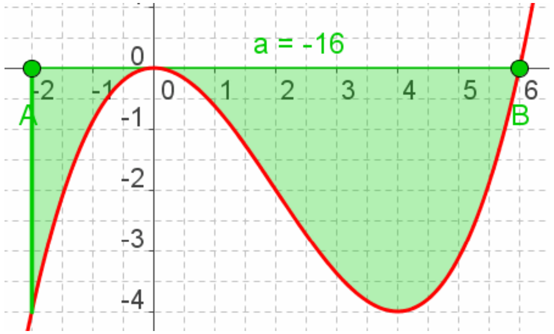


$$\int_a^b f(x)dx$$

Intervall [A,B]

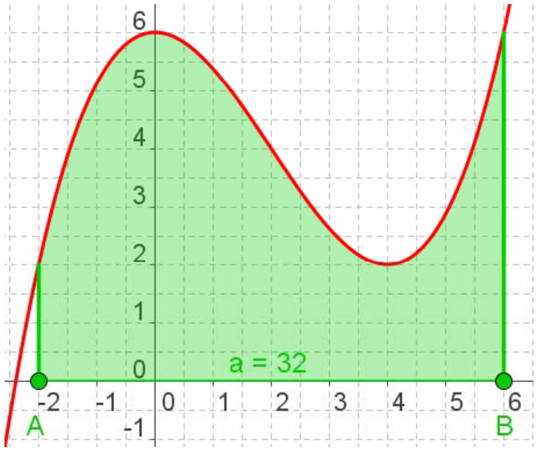


Das Integral ist eine Flächenbilanz mit negativen und positiven Flächen.



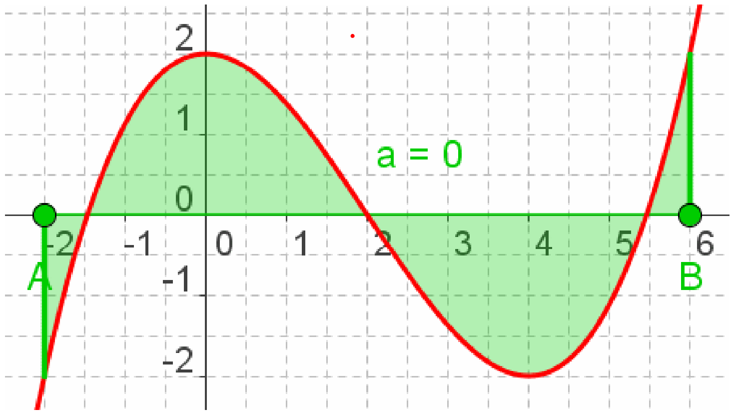
Sind die Werte von f im ganzen Intervall negativ, dann ist auch das Integral negativ.

Properties of the Integrals

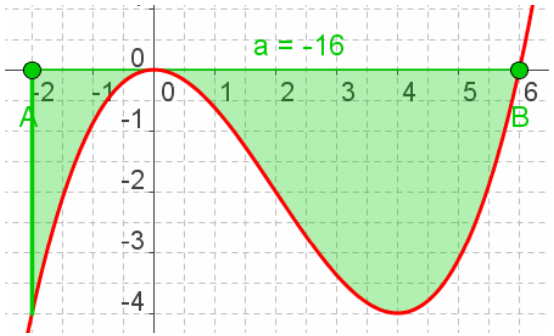


$$\int_a^b f(x)\,dx$$

interval [A,B]

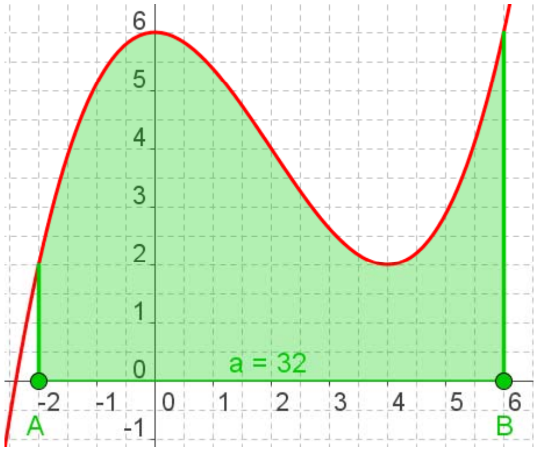


The integral is a balance of areas with negative and positive values.



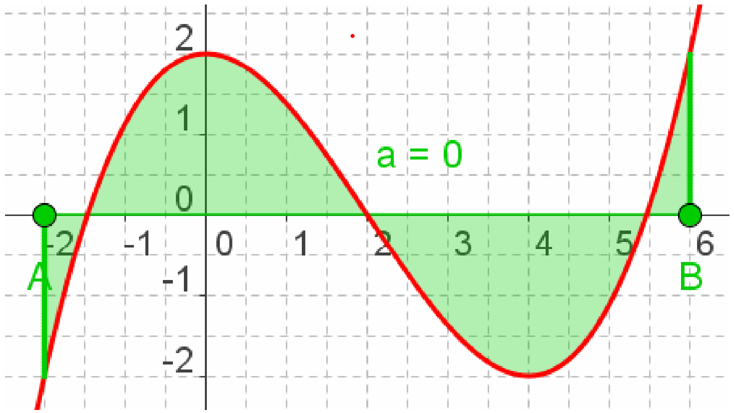
If the values of f are negative in the whole interval than the integral is negativ.

Eigenschaften des Integrals

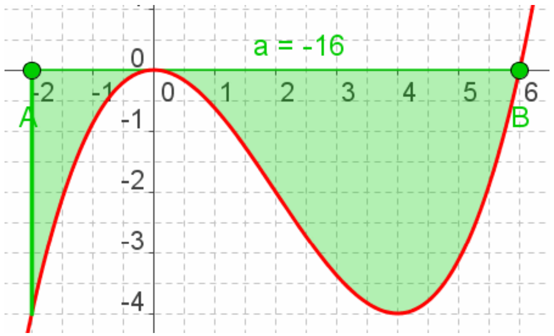


$$\int_a^b f(x)dx$$

Intervall [A,B]



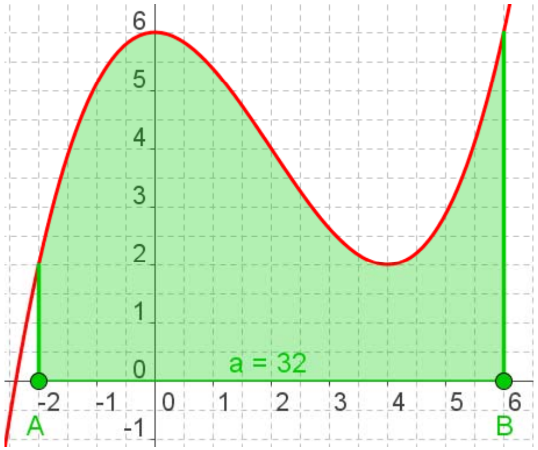
Das Integral ist eine Flächenbilanz mit negativen und positiven Flächen.



Beim Vertauschen der Grenzen ändert sich das Vorzeichen des Integrals

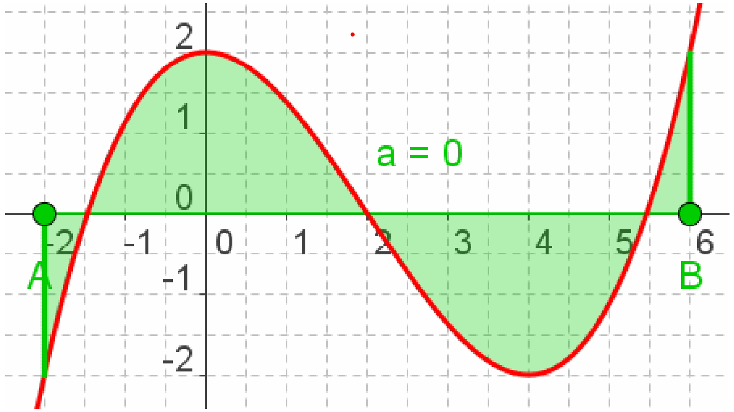
Sind die Werte von f im ganzen Intervall negativ, dann ist auch das Integral negativ.

Properties of the Integrals

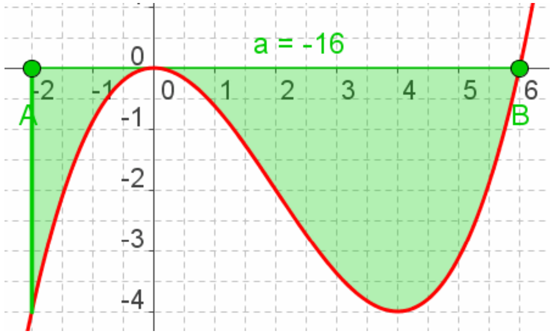


$$\int_a^b f(x)dx$$

interval [A,B]



The integral is a balance of areas with negative and positive values.



By changing the borders
the sign
of the integral changes.

If the values of f are negative in the whole interval
than the integral is negativ.

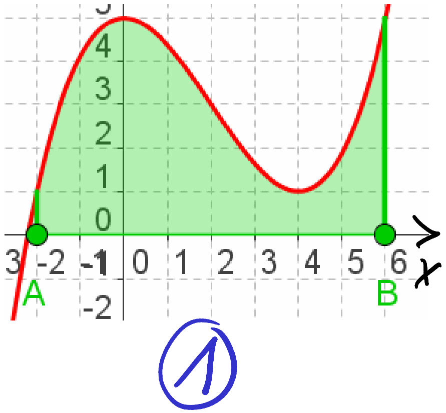
Übungen zum Integral

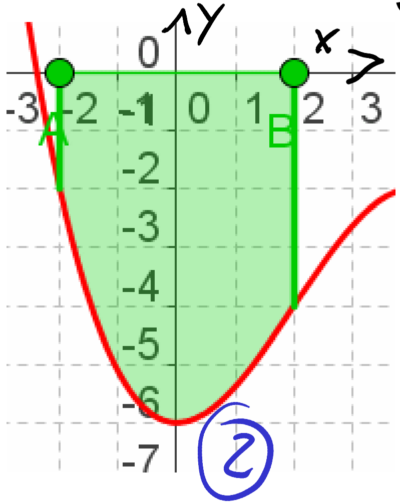
$$\int_a^b f(x)d\,x$$

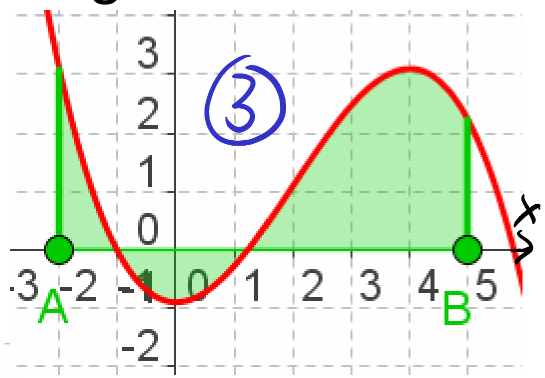
Intervall [A,B]

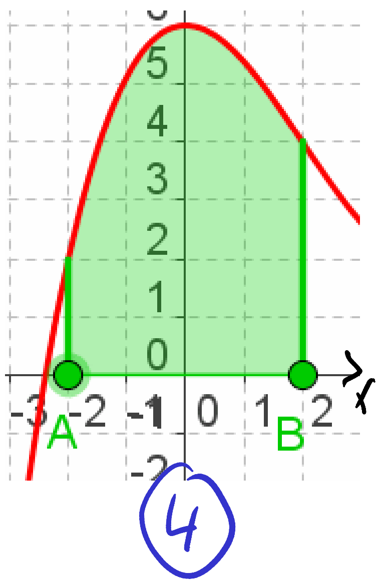
$\pm 8, \pm 20, \pm 24$

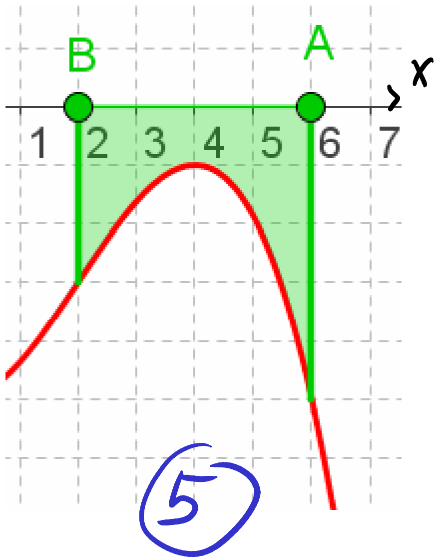
mögliche Werte

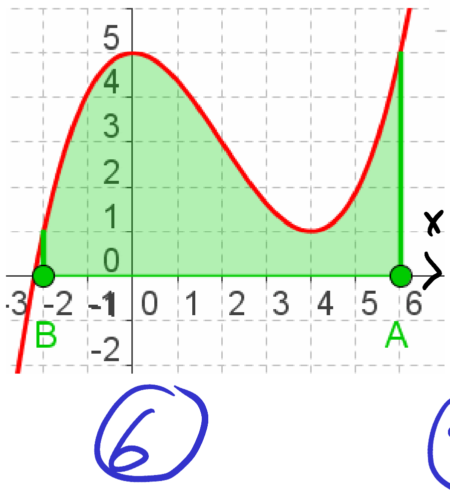


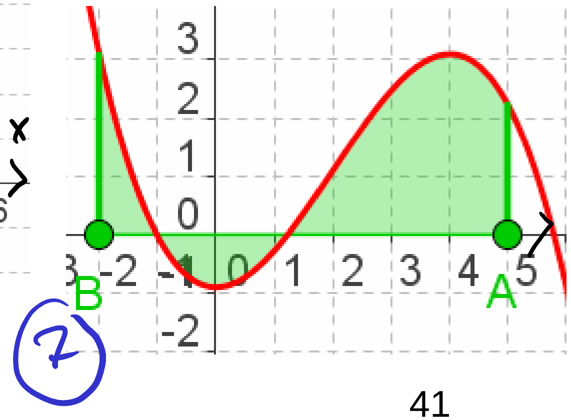








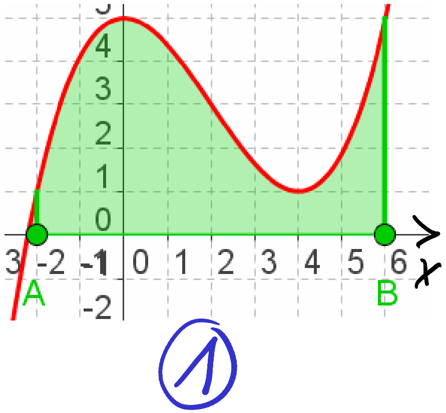


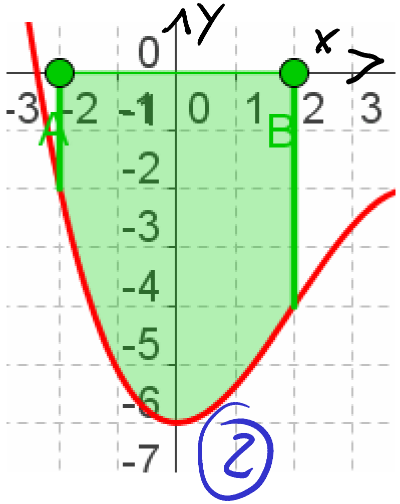


Exercise with the Integral

$$\int_a^b f(x)d\,x$$

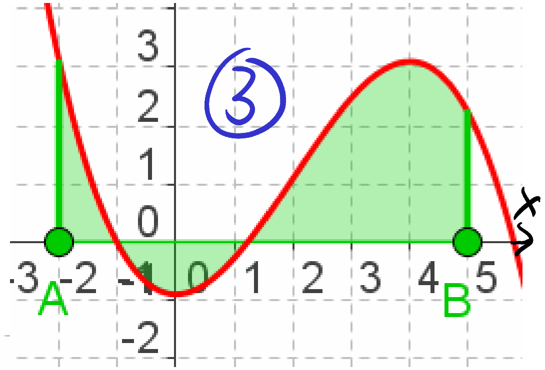
interval [A,B]

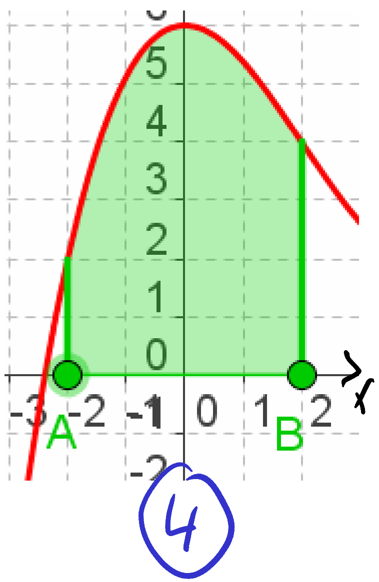


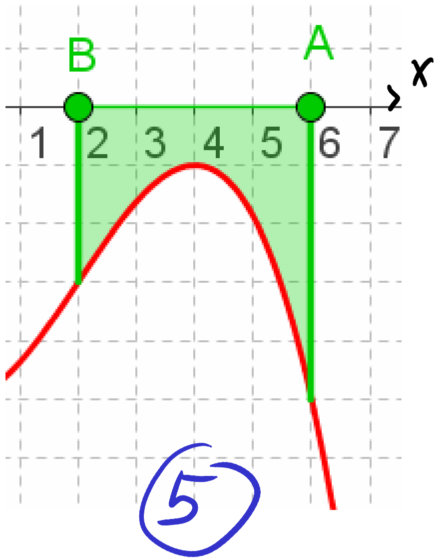


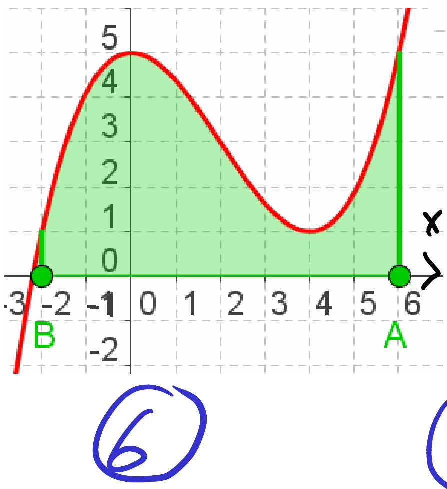
$$\pm 8, \pm 20, \pm 24$$

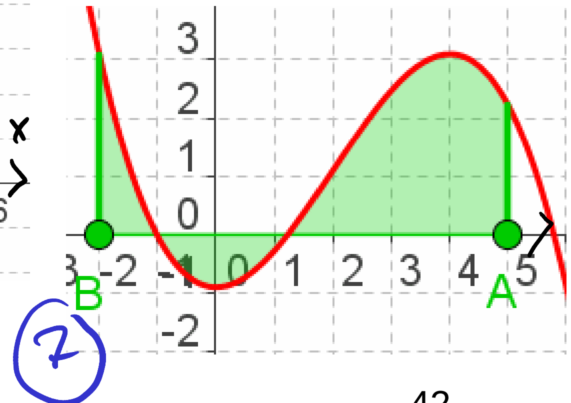
possible values











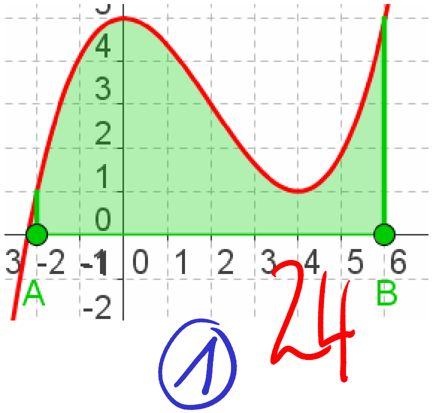
Übungen zum Integral

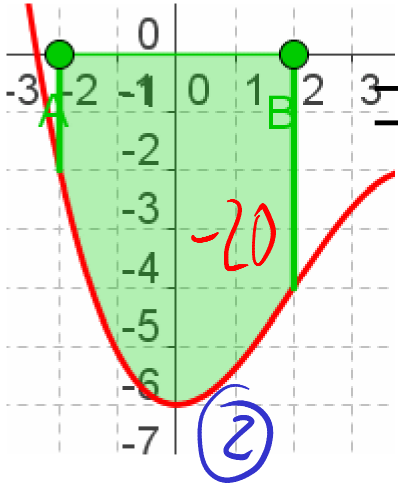
$$\int_a^b f(x)dx$$

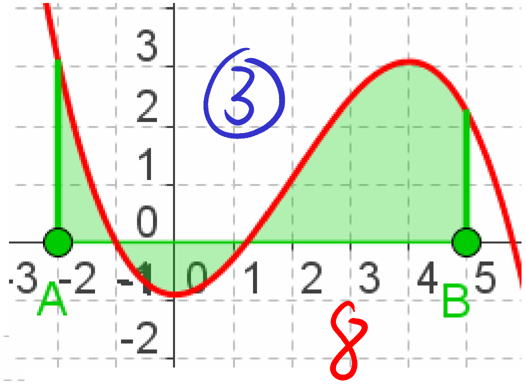
Intervall [A,B]

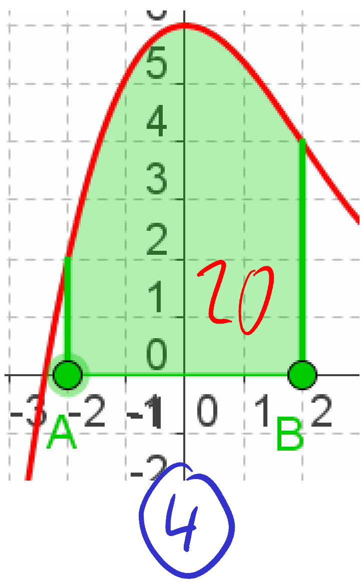
$\pm 8, \pm 20, \pm 24$

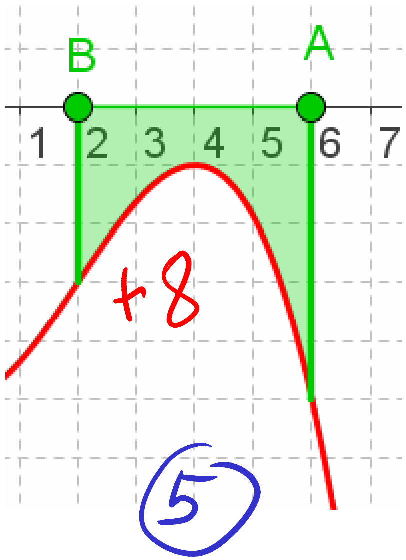
mögliche Werte

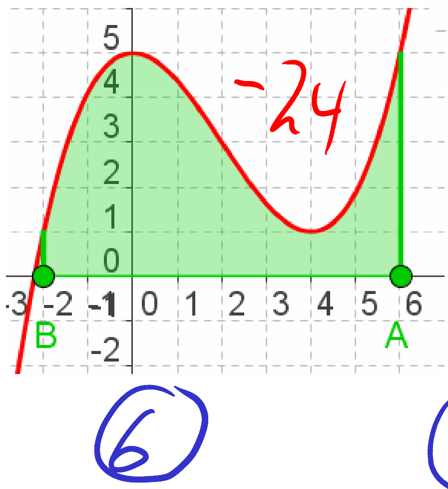


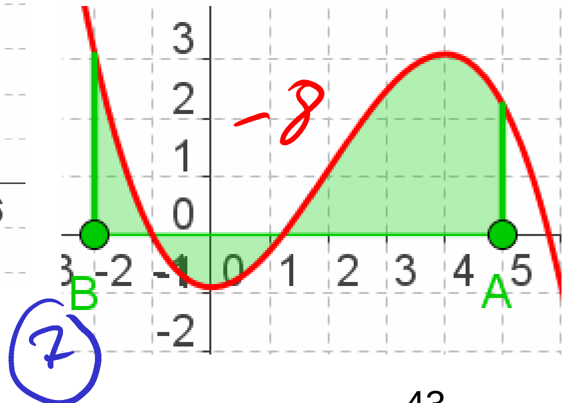








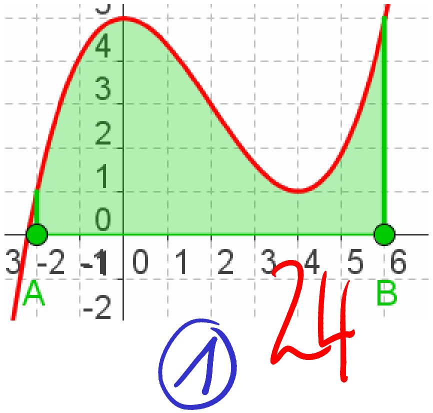


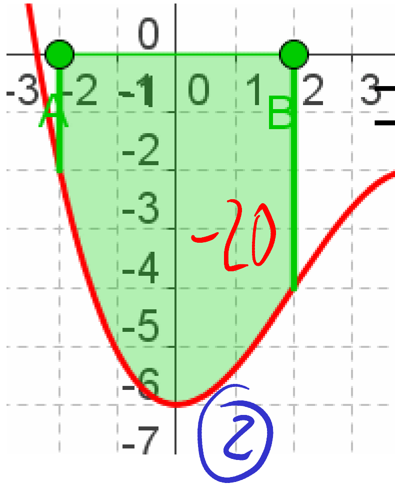


Exercise with the Integral

$$\int_a^b f(x)dx$$

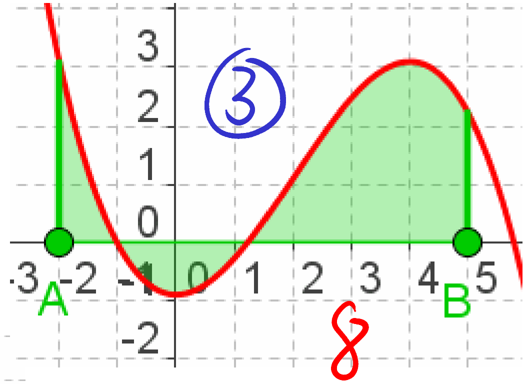
interval [A,B]

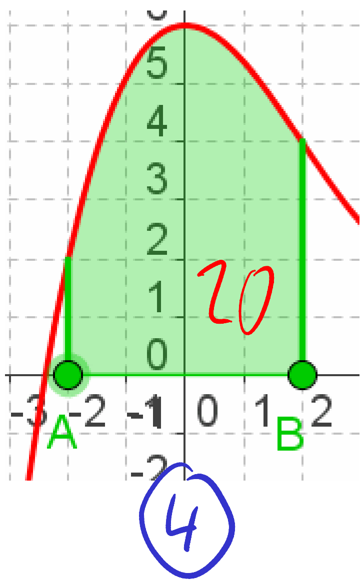


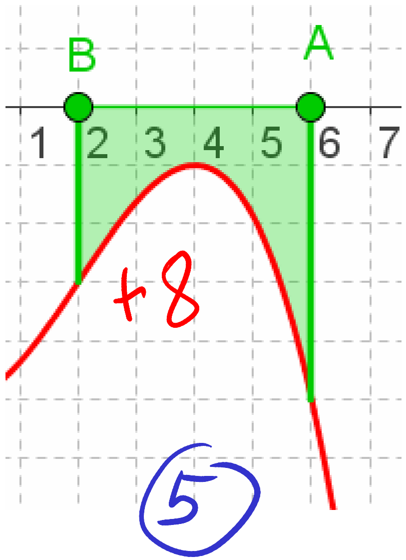


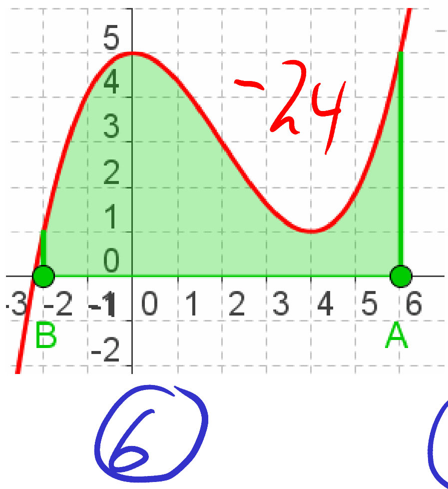
$$\pm 8, \pm 20, \pm 24$$

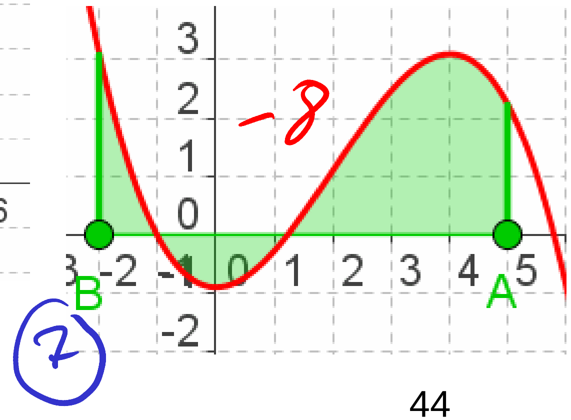
possible values









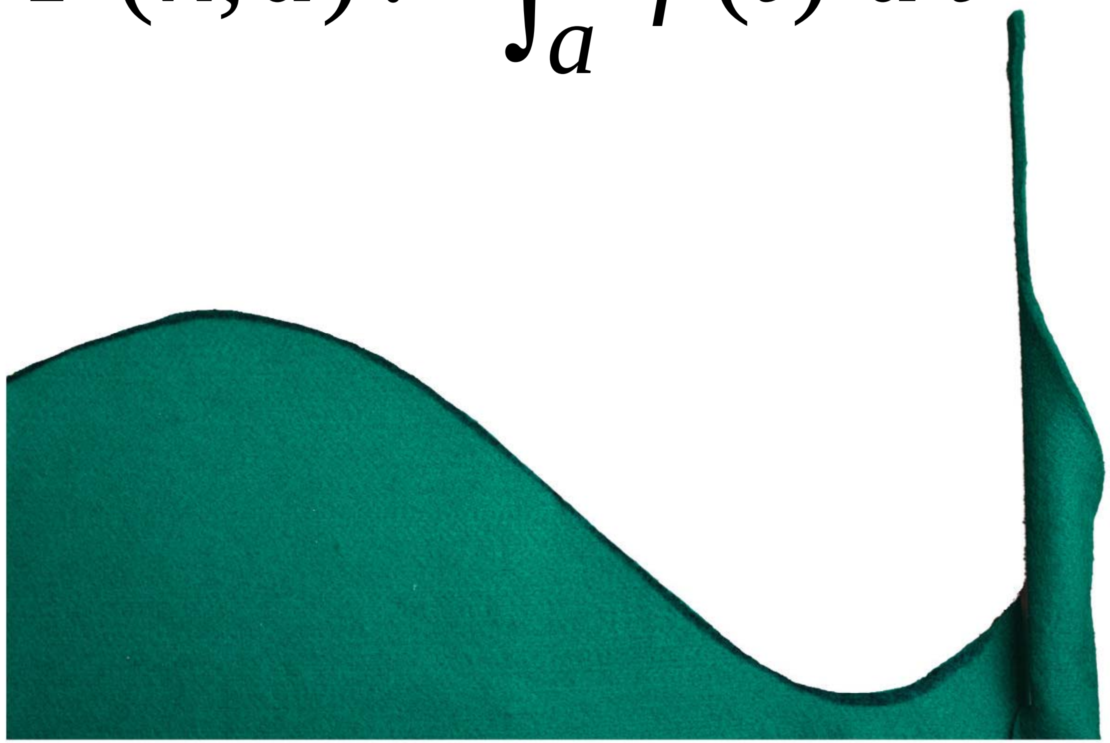


Die Integralfunktion

„Teppich-Abroll-Funktion“ $$F(x,a) := \int_a^x f(t)\, dt$$

a

x



a

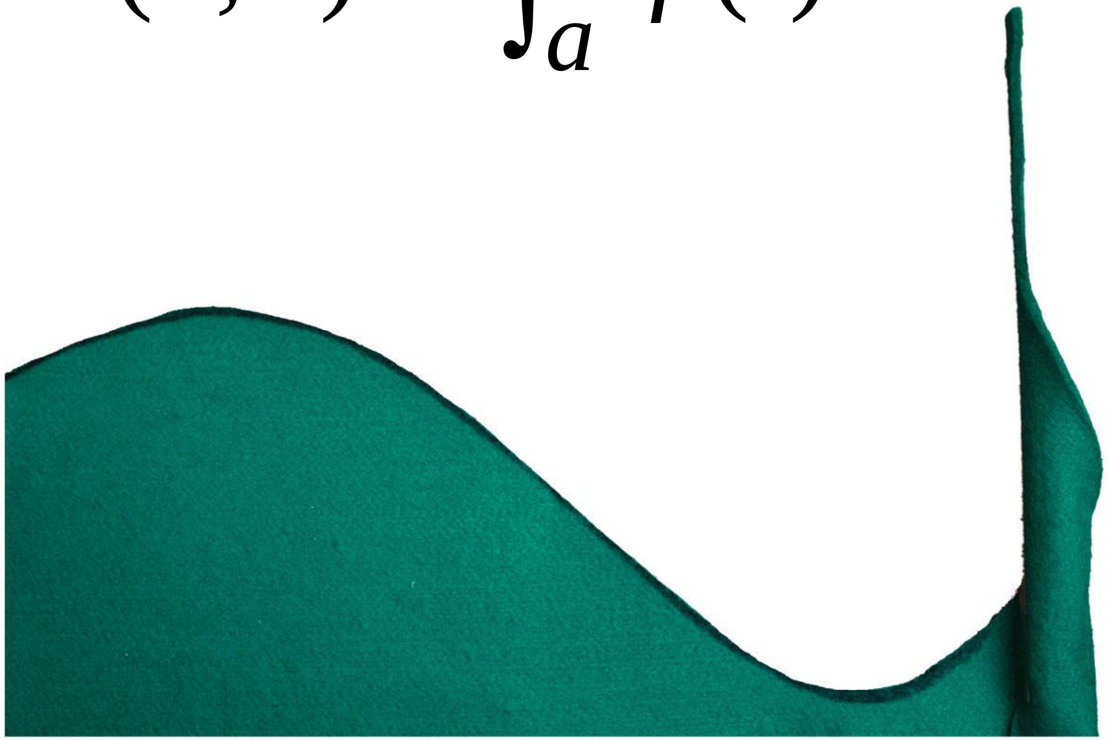
x

The Integral Funktion

„carpet scrolling funktion“

$$F(x,a) := \int_a^x f(t)\, dt$$

a x



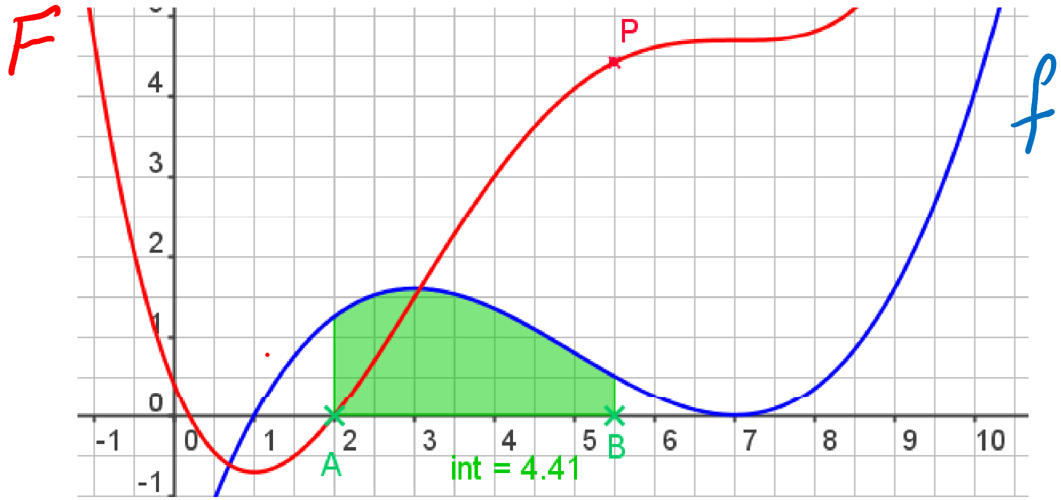
a x

Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2103 http://www.leuphana.de/matheomnibus

Die Integralfunktion

$$F(x,a) := \int_a^x f(t)\,dt$$

„Teppich-Abroll-Funktion“

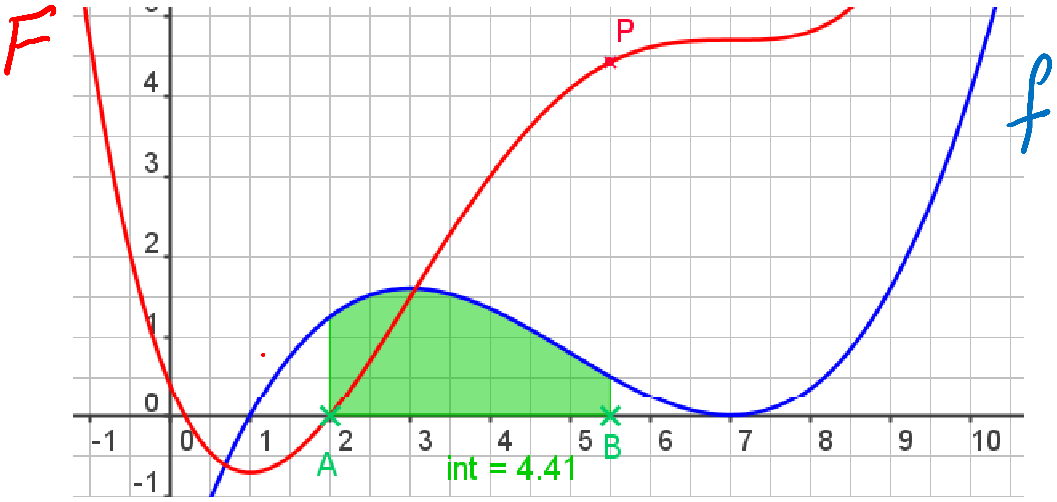


Ordinate von P zeigt die abgerollte Fläche an.

The Integral Funktion

$$F(x,a) := \int_a^x f(t)\,dt$$

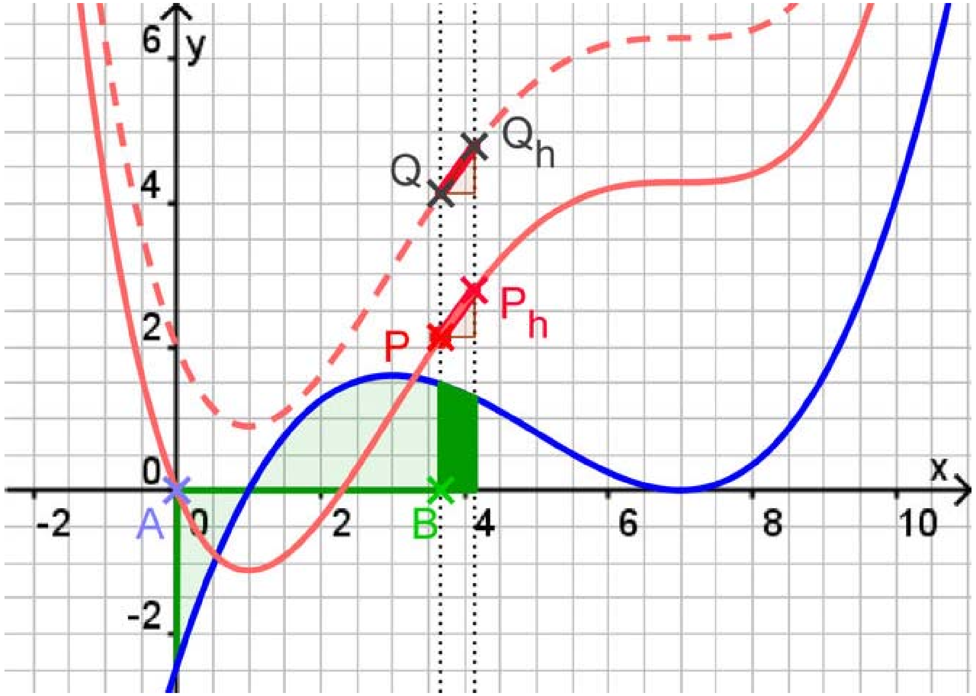
„carpet scrolling funktion“



The ordinate of P shows the scrolled area.

Die Integralfunktion

$$F(x,a) := \int_a^x f(t)\,dt$$

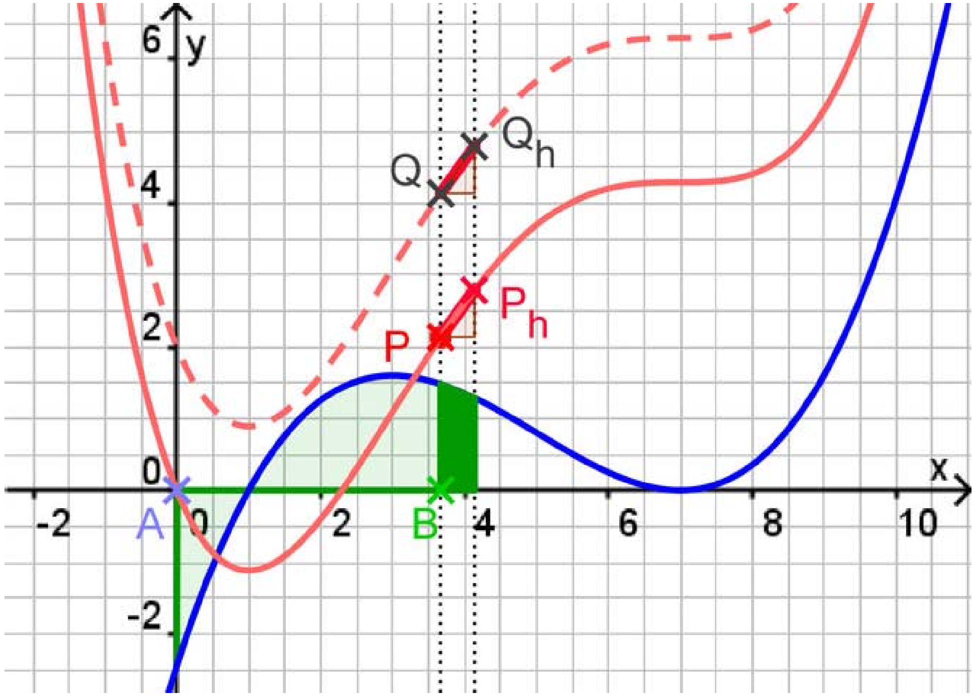


Der Zuwachs der Integralfunktion hängt nur vom Zuwachs der Fläche ab. Also sind die verschiedenen Integralfunktionen an jeder Stelle x **gleich steil**.

(x ist hier die Stelle von B)

The Integral Funktion

$$F(x,a) := \int_a^x f(t)\,dt$$

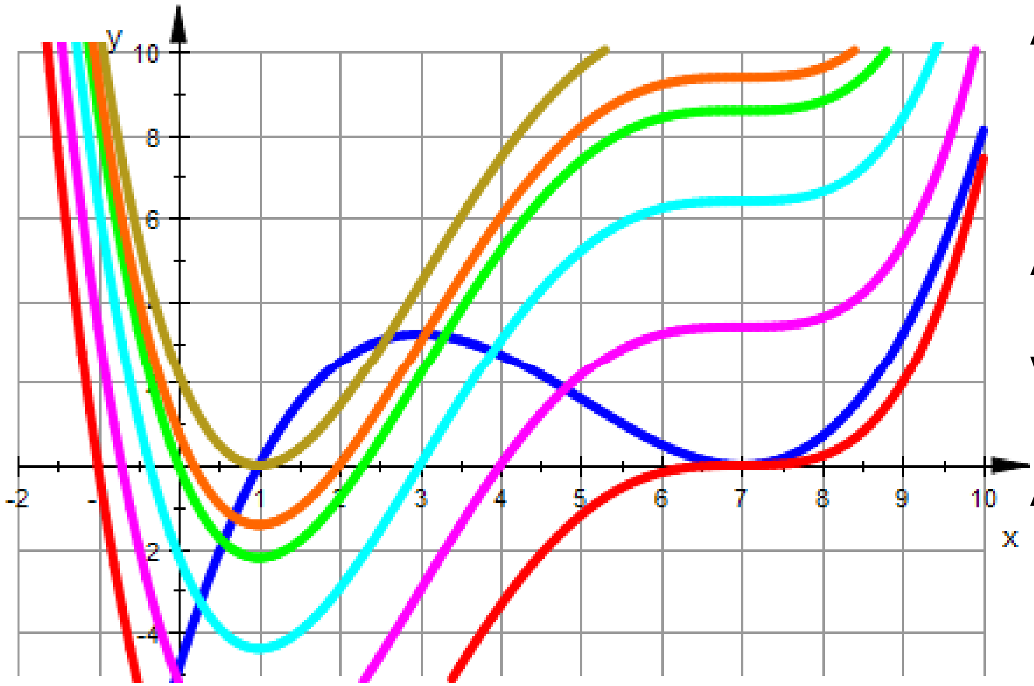


The growth of the integral-function depends only on the growth of the area. Therefore all the different integral functions have in every position x **the same slope**.

(here x is the position of B)

Die Integralfunktion

$$F(x,a) := \int_a^x f(t)\,dt$$



Alle Integralfunktionen

haben dieselbe Form.

An den Extremstellen

von F hat f eine Nullstelle.

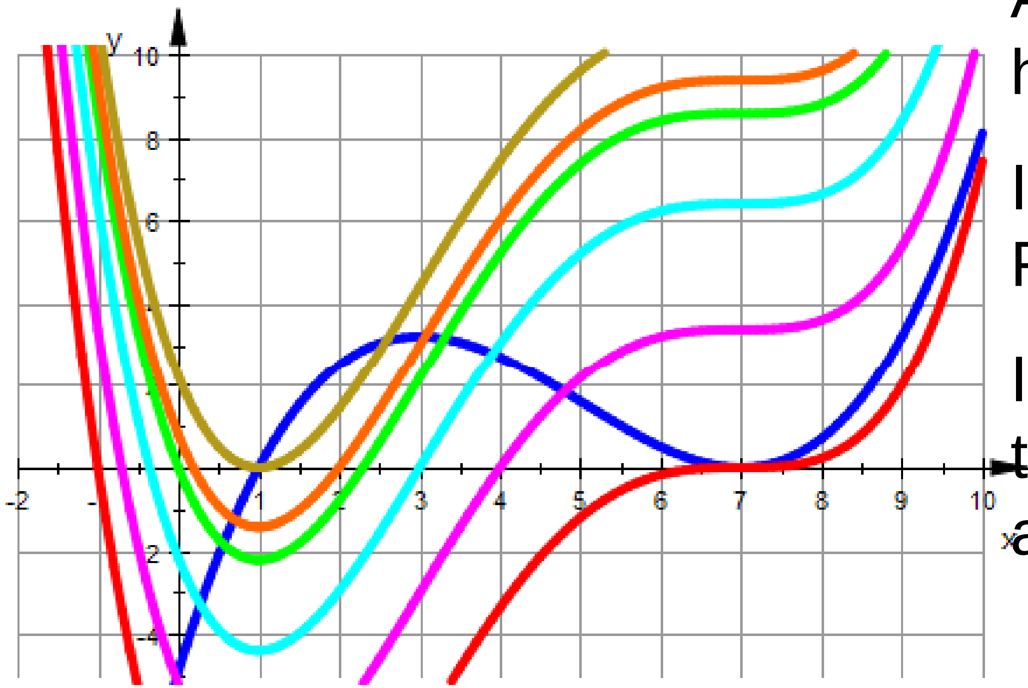
An der Sattelstelle von F

hat f eine Berühr-Nullstelle.

Wo F eine Wendestelle hat, hat f eine Extremstelle.

The Integral Funktion

$$F(x,a) := \int_a^x f(t)\,dt$$



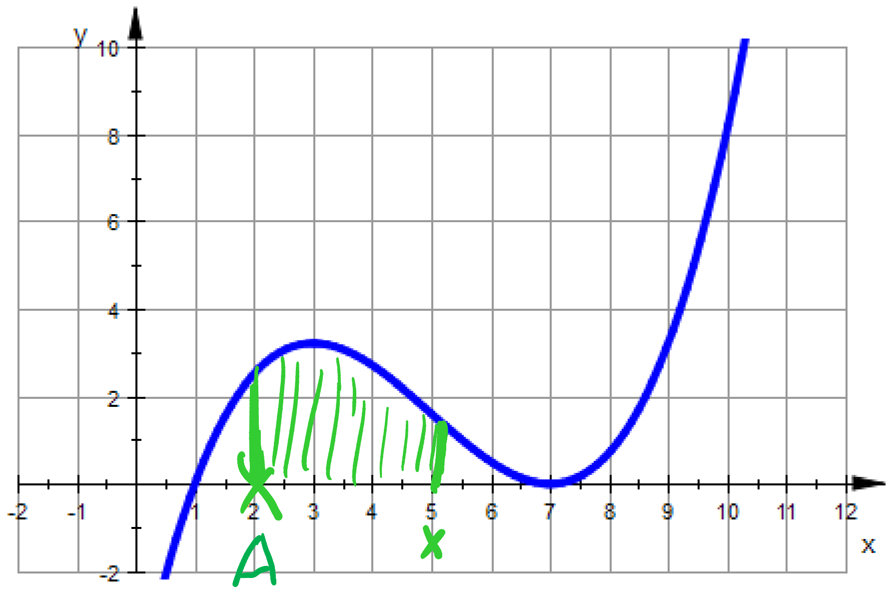
All integral functions have the same form.

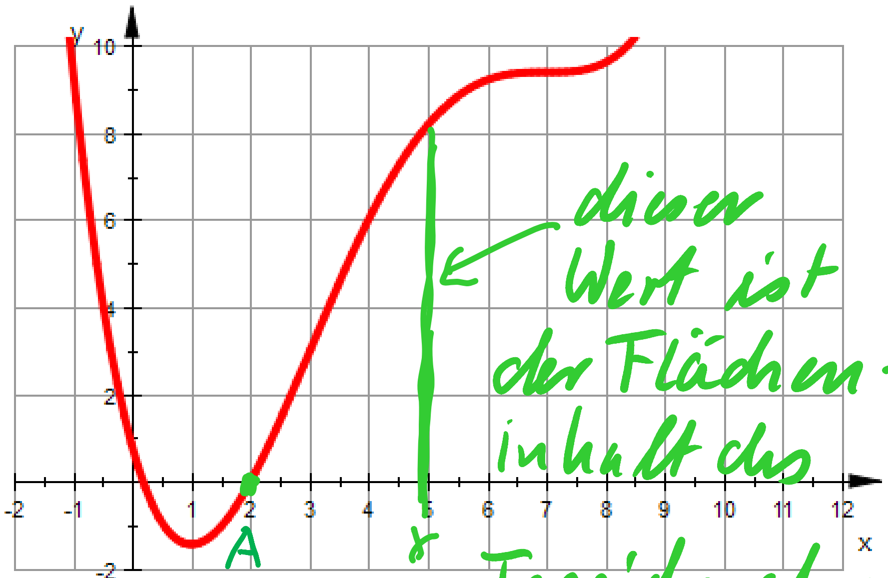
In the extrem abscissas of F the function f has a zero.

In the saddle-abscissa of F the function f has the x-axis as a tangent.

In the position of inflection of F there is an extreme position of f.

Nochmal die Teppichabrollfunktion





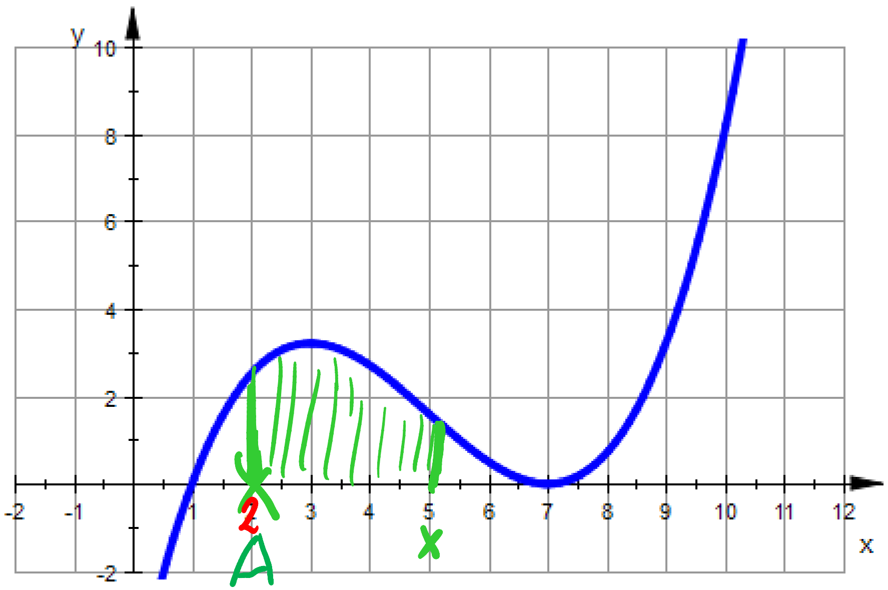
dieser Wert ist der Flächeninhalt des Teppichs, abgerollt bis $x=5$

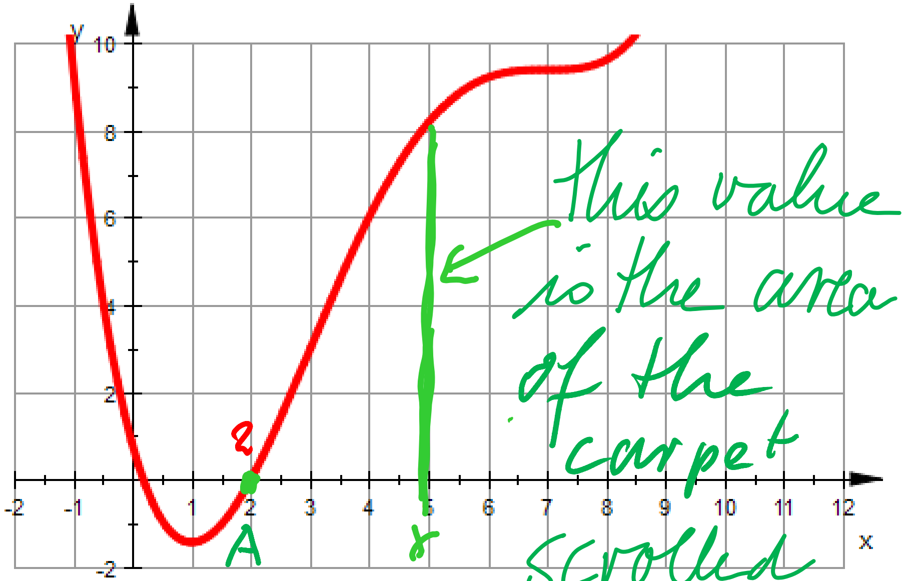
f als Rand des Teppichs

$$F(x) = \int_2^x f(t)\,dt$$

Integralfkt zum Start 2

Once Again the Carpet scrolling funktion





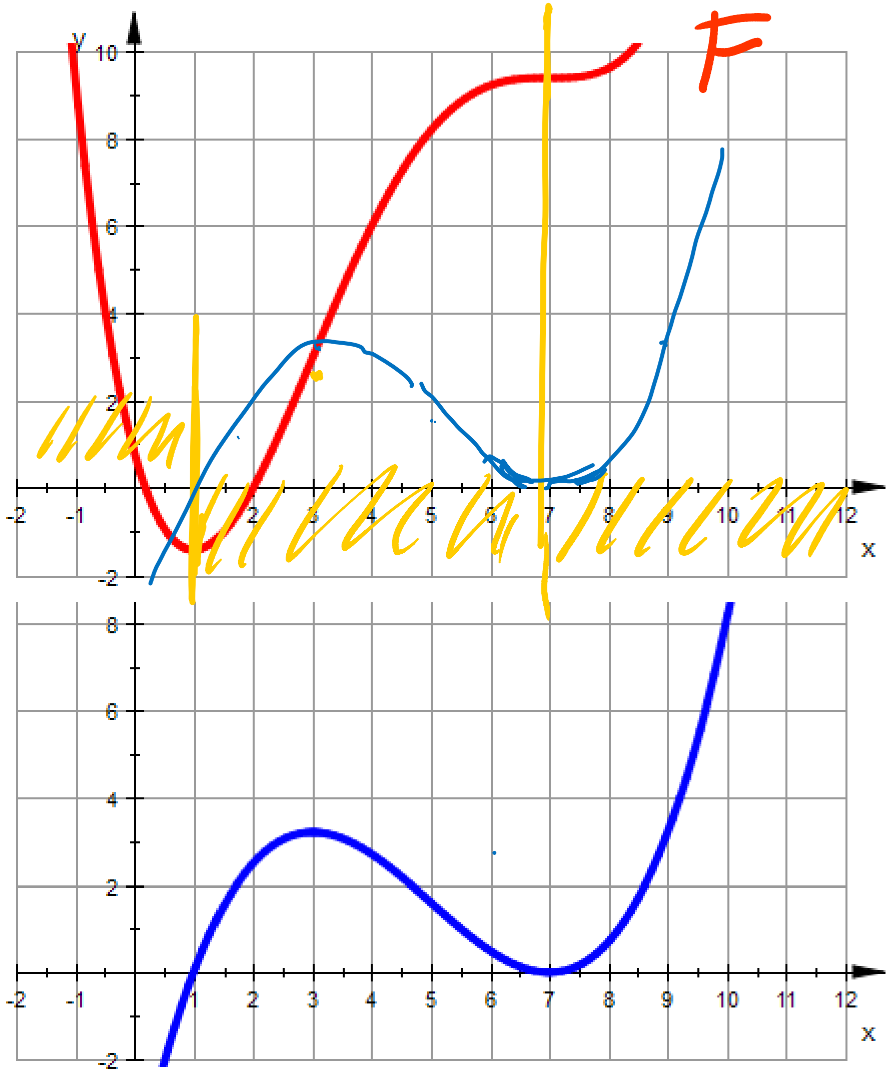
this value is the area of the carpet scrolled until $x=5$

f as the border of the carpet

$$F(x) = \int_2^x f(t)\,dt$$

integral-function will start in 2

Die Intergralfunktion F von f =„Teppichabrollfunktion“

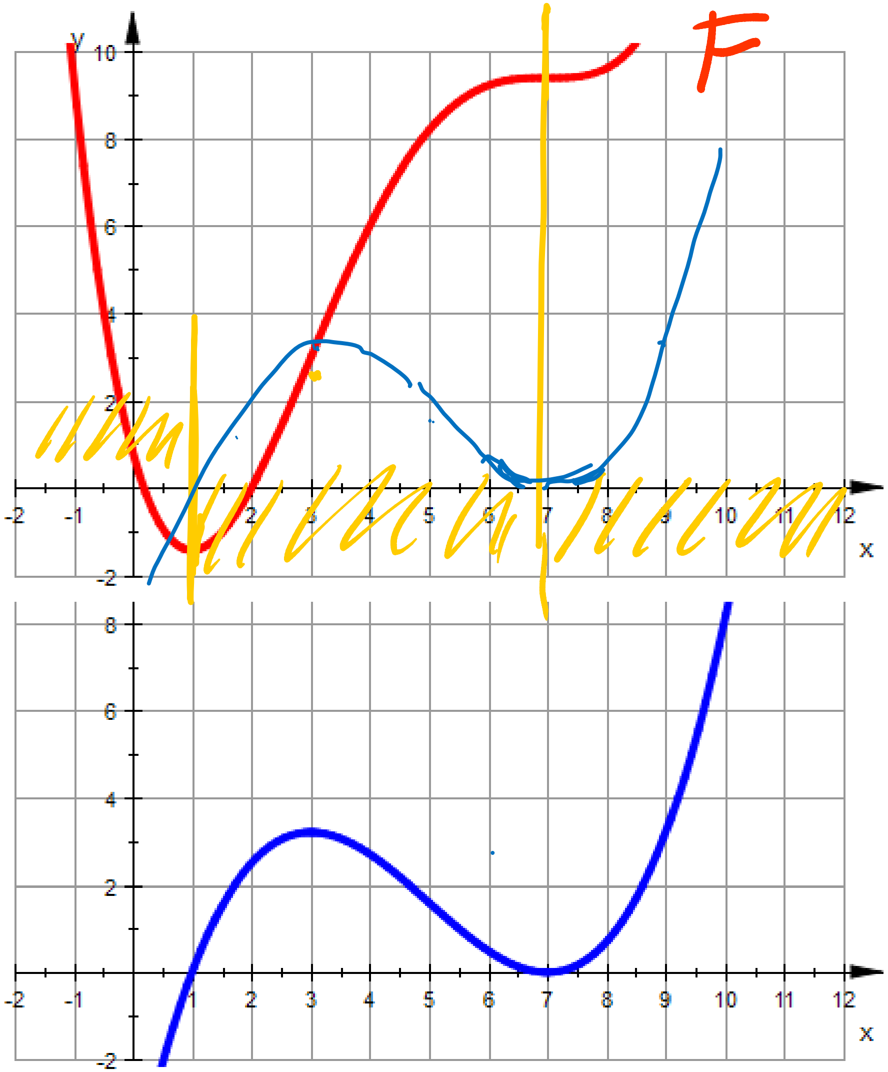


f als Ableitung von F

dasselbe!

f als Randfkt. für den abgerollten Teppich

The Integral Function F von f =„ Carpet scrolling funktion“



f as the derivative of F

it's the same!

f as the border of the scrolled carpet

Hauptsatz der Differential- und Integralrechnung

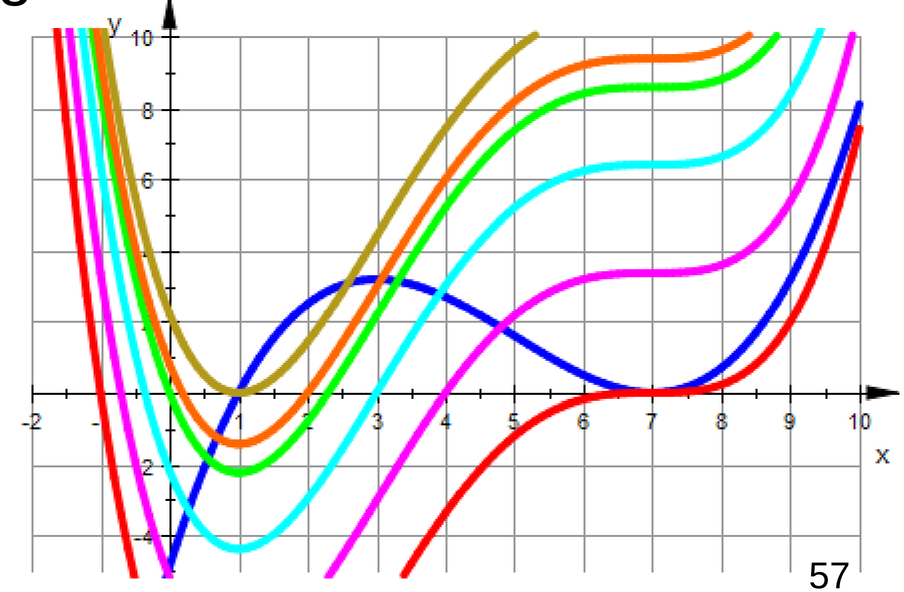
$$\boxed{f(x) = F'(x)}$$

d. h. Alle Integralfunktionen F zu f mit beliebigem Start

haben ihr f auch als Ableitung. Sie heißen daher auch

„Stammfunktionen“ von f,

f blau



sie unterscheiden sich nur um eine

additive Konstante c. Man schreibt:

$$F(x) = \int f(x)\,dx + c$$

Principal Theorem of the Calculus

$$\boxed{f(x) = F'(x)}$$

That is: All integral functions F of f with arbitrary start have their own f as their derivative. For that we call them „antiderivative" von f.

f blue

All possible F differ only in an additive constant c. One write:

$$F(x) = \int f(x)\,dx + c$$