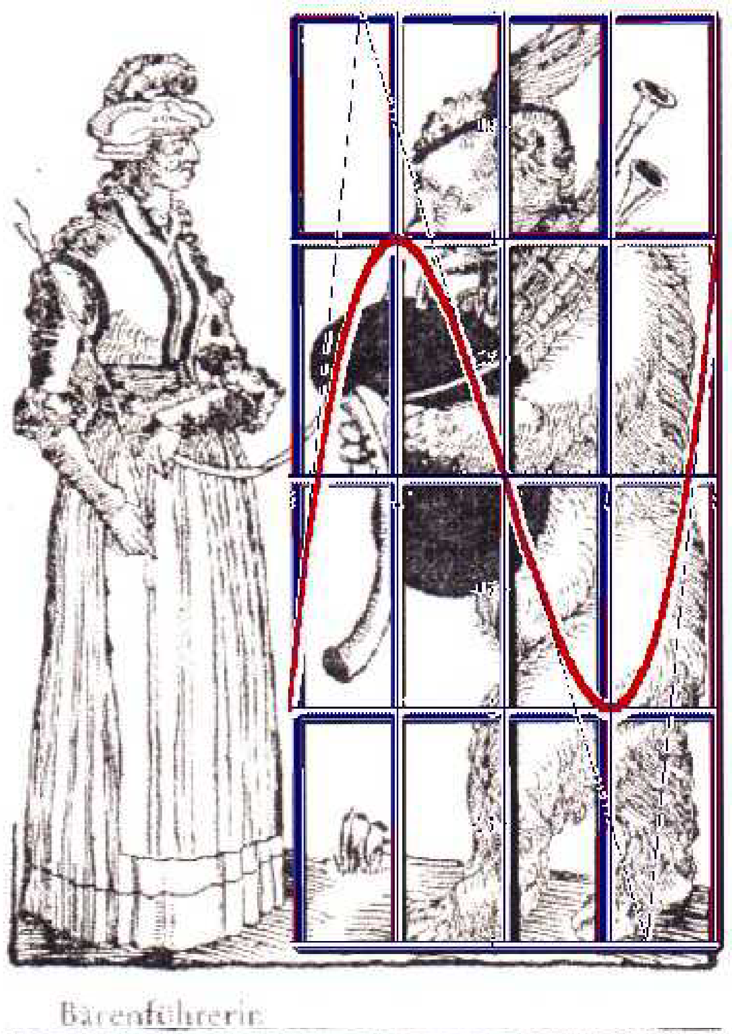
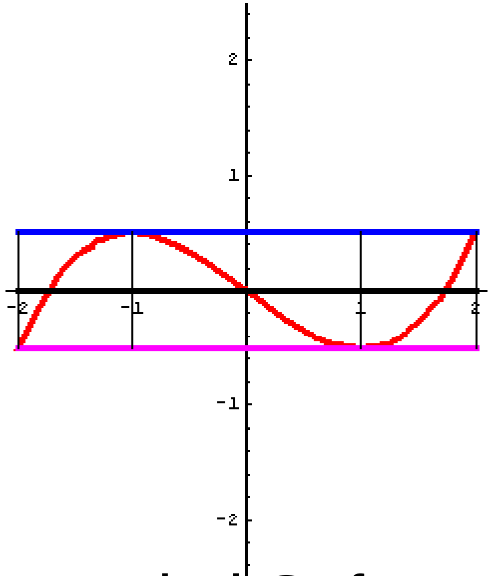
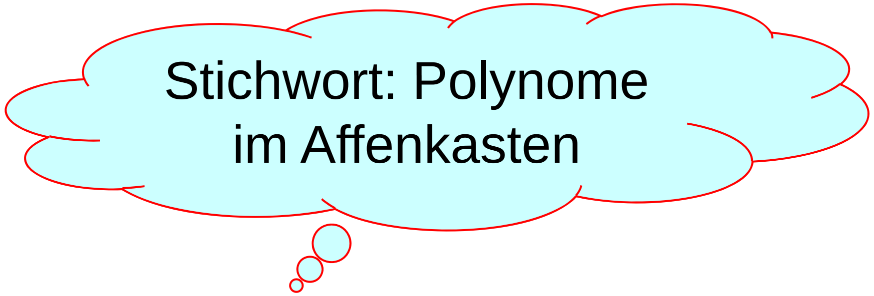
Polynome und mehrfache Nullstellen





Polynome sind Gefangene ihrer leicht durchschaubaren Eigenschaften.



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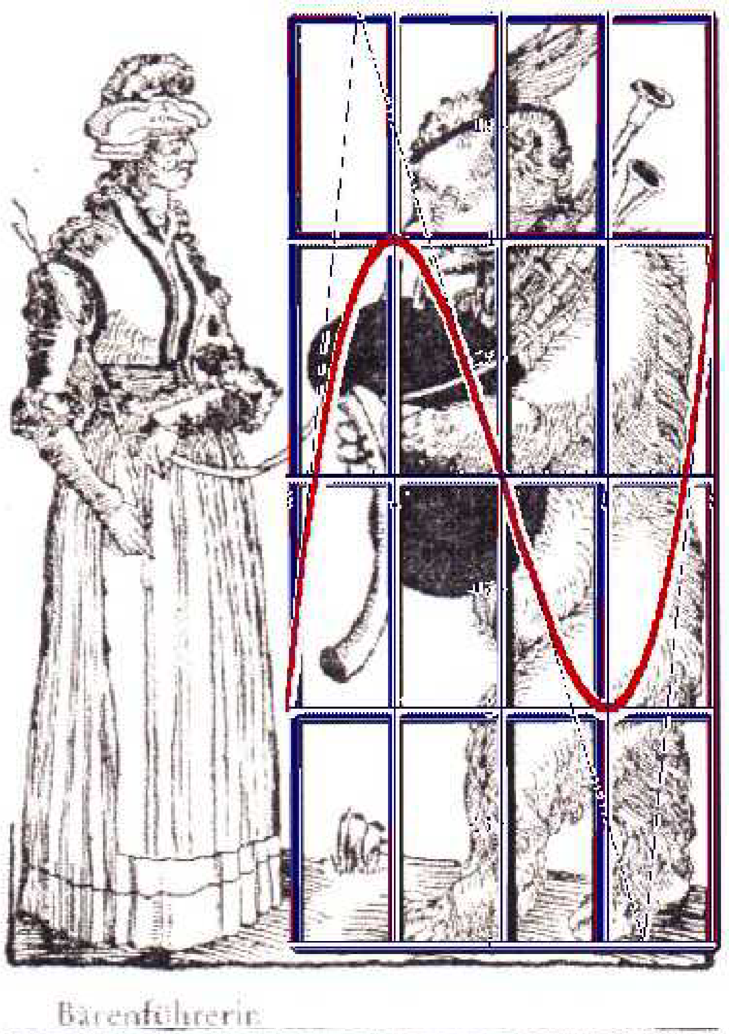
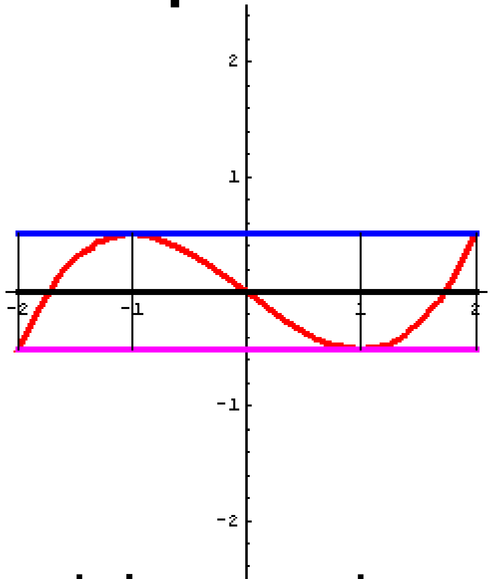
Polynomials and Multiple Zeros





polynomials are prisons of their obvious characteristics

These are named: polynomials in an „arp box“

key word: Polynome im Affenkasten

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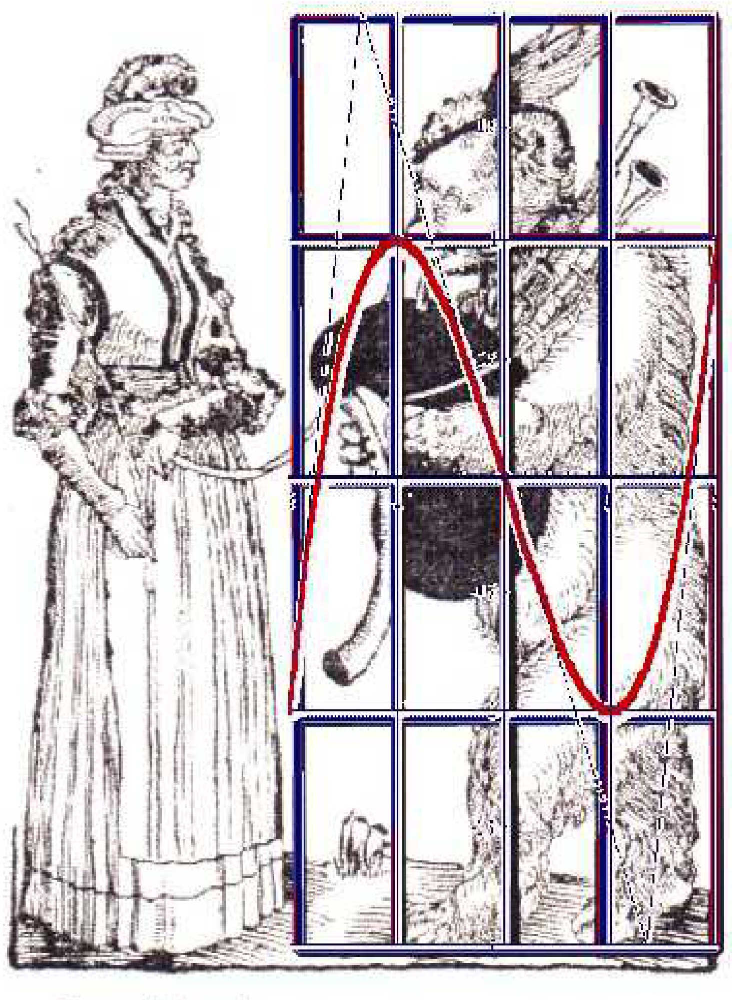
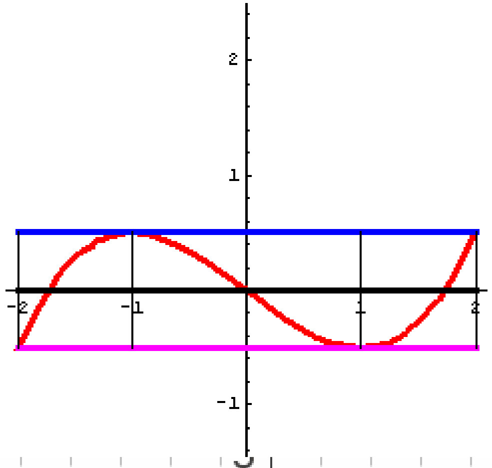
Polynome und mehrfache Nullstellen

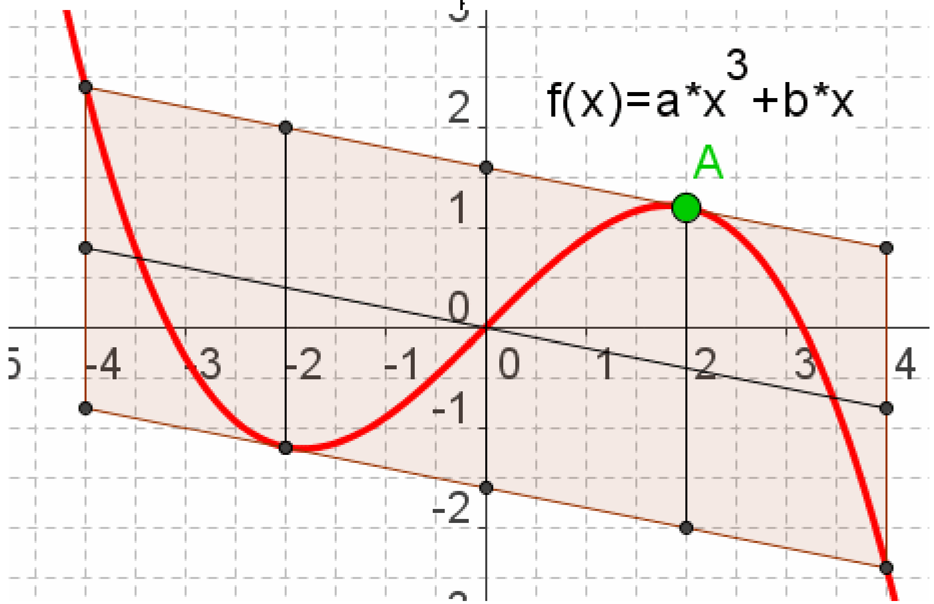


Bärenführerin



gerade

schräg



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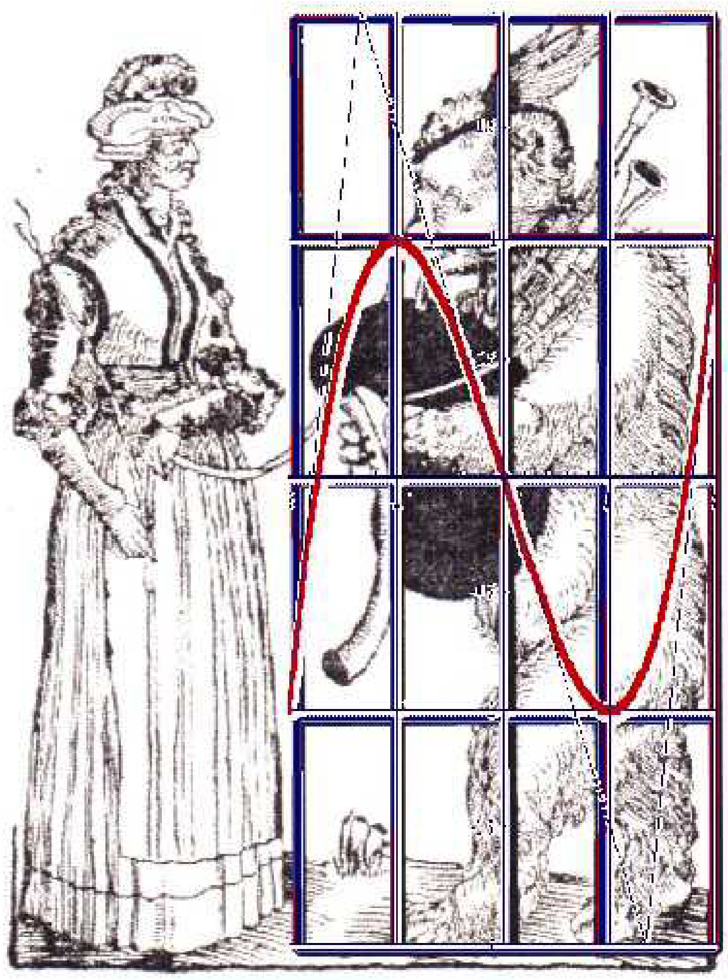
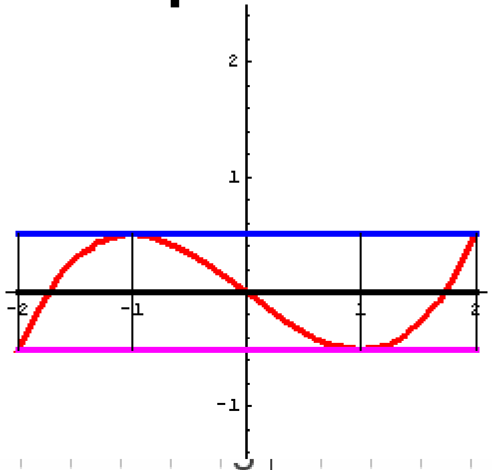
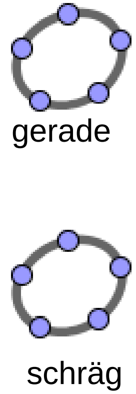
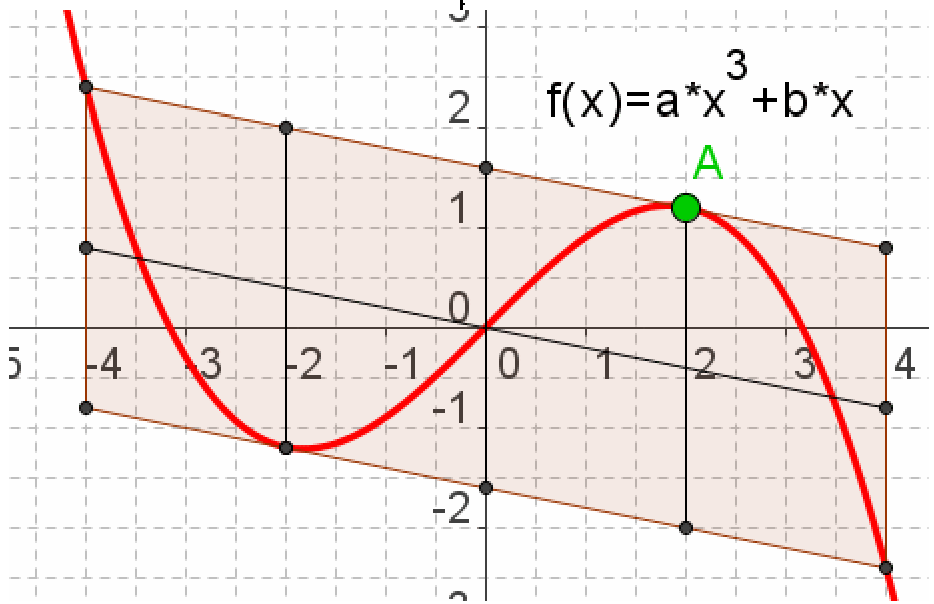
Polynomials and Multiple Zeros



Bärenführerin







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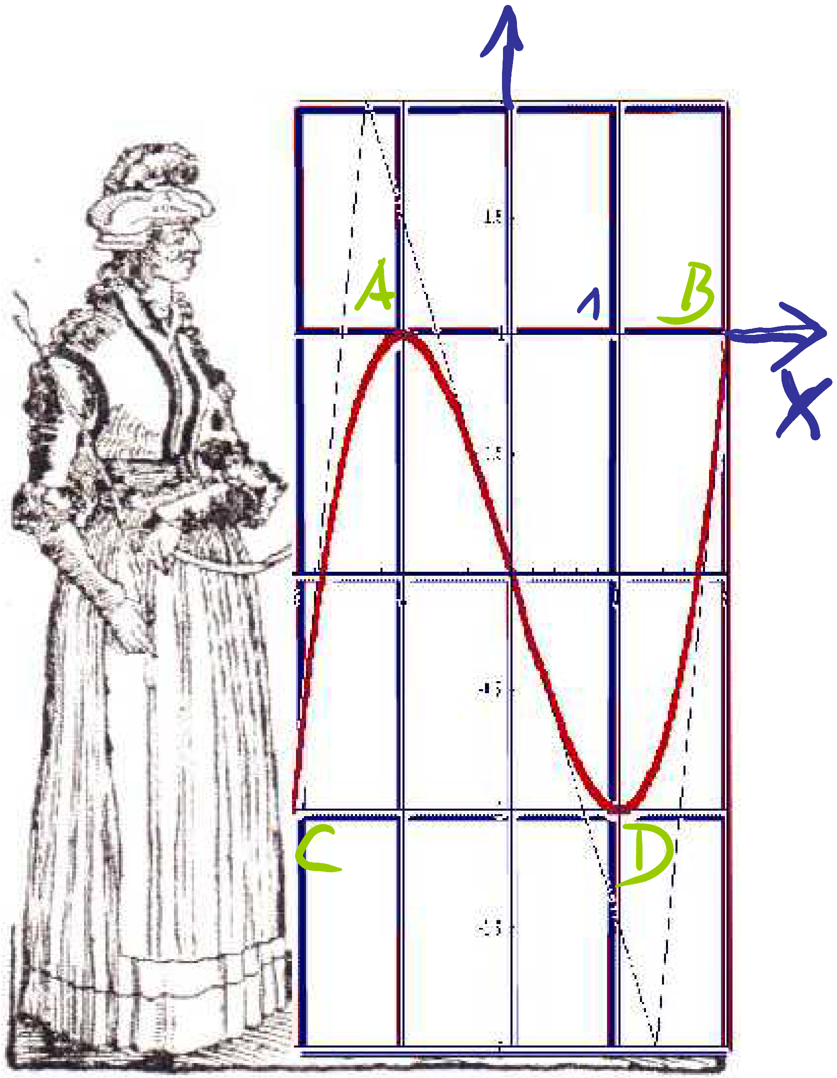
Nullstellen ⟺ Linearfaktoren

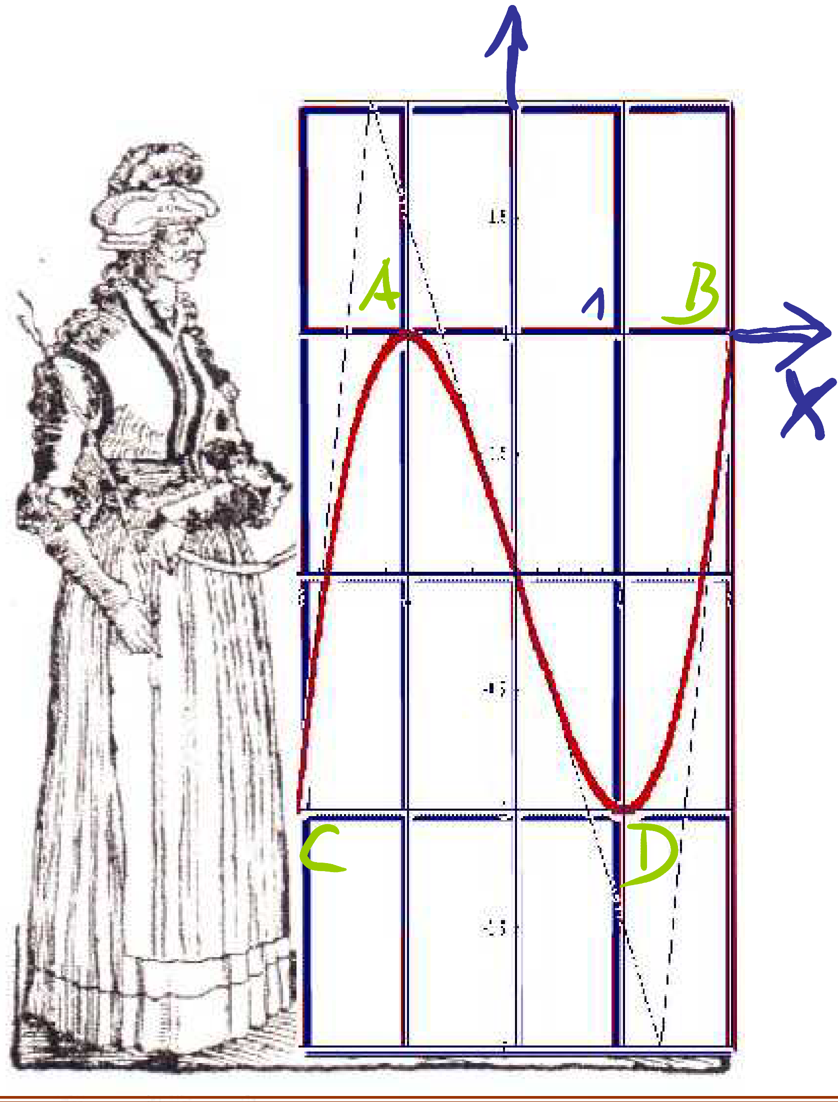
Lin.



$f(x) = t\,(x+1)^2\,(x-2)$

double zero	single zero
x= -1	x= 2
A	B

Zeros ⟺ Linear Factors

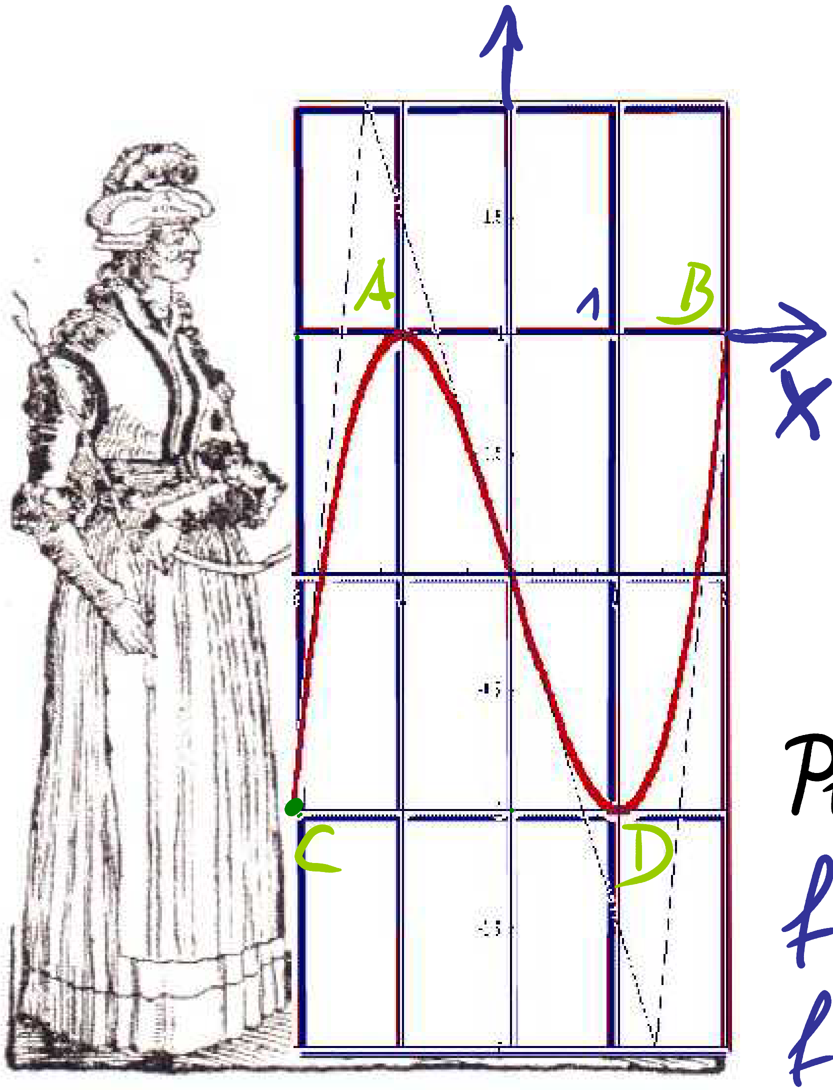


Lin.

$f(x) = t\,(x+1)^2\,(x-2)$

doppelte Nullstelle	einfache Nullstelle
x= -1	x= 2
A	B

Nullstellen $\Longleftrightarrow$ Linearfaktoren



$f(x) = (x+1)^2 (x-2)$

doppelte Nullstelle	einfache Nullstelle
x= -1	x= 2
A	B

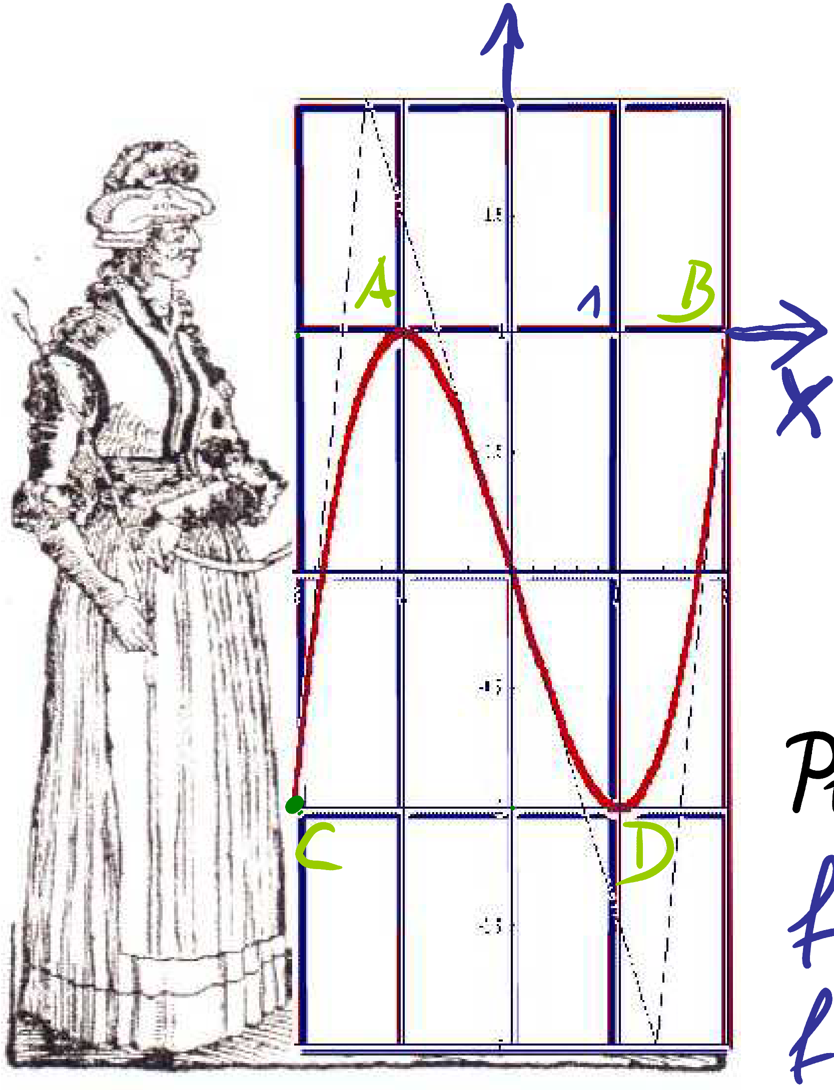
Proben

$f(-2) = (-2+1)^2(-2-2)$ C

$= -4$ "

$f(1) = (1+1)^2(1-2) = -4$ D

Zeros ⟺ Linear Factors



$f(x) = (x+1)^2(x-2)$

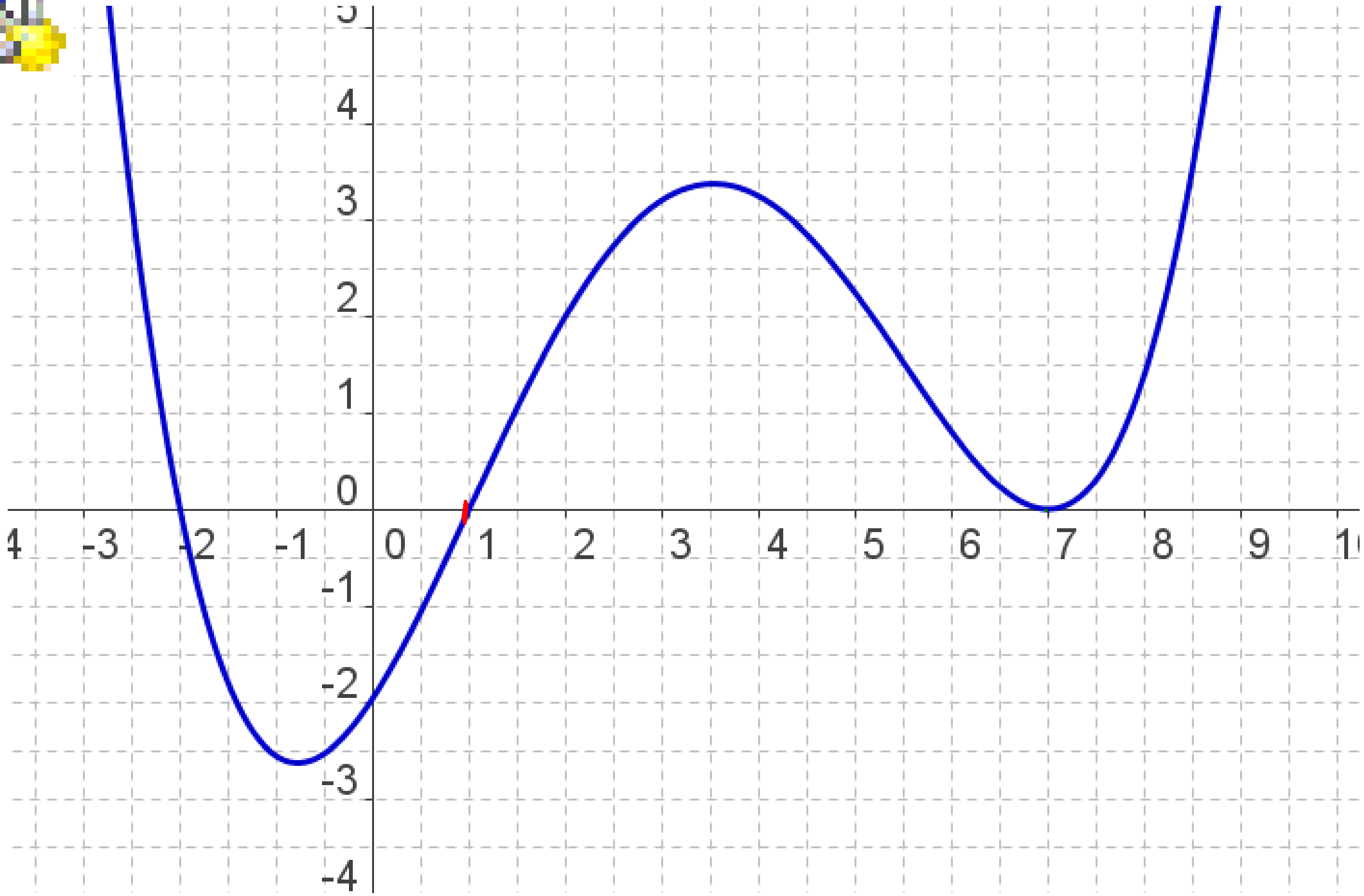
double zero	single zero
x= -1	x= 2
A	B

Proben

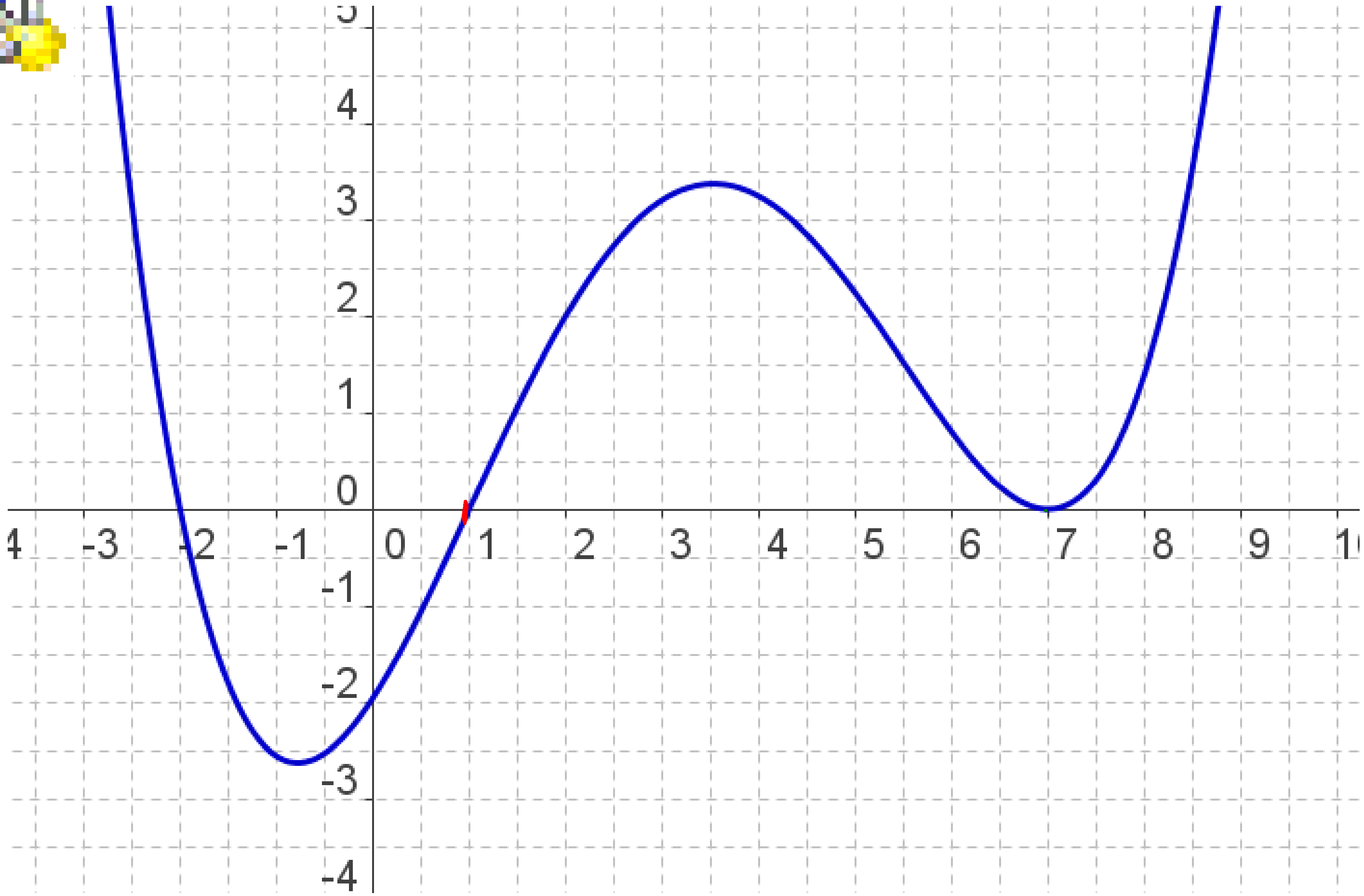
$f(-2) = (-2+1)^2(-2-2) = -4$ C

$f(1) = (1+1)^2(1-2) = -4$ D

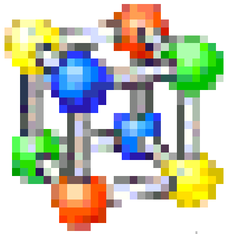
Welche Gleichung kann dieses Polynom haben?

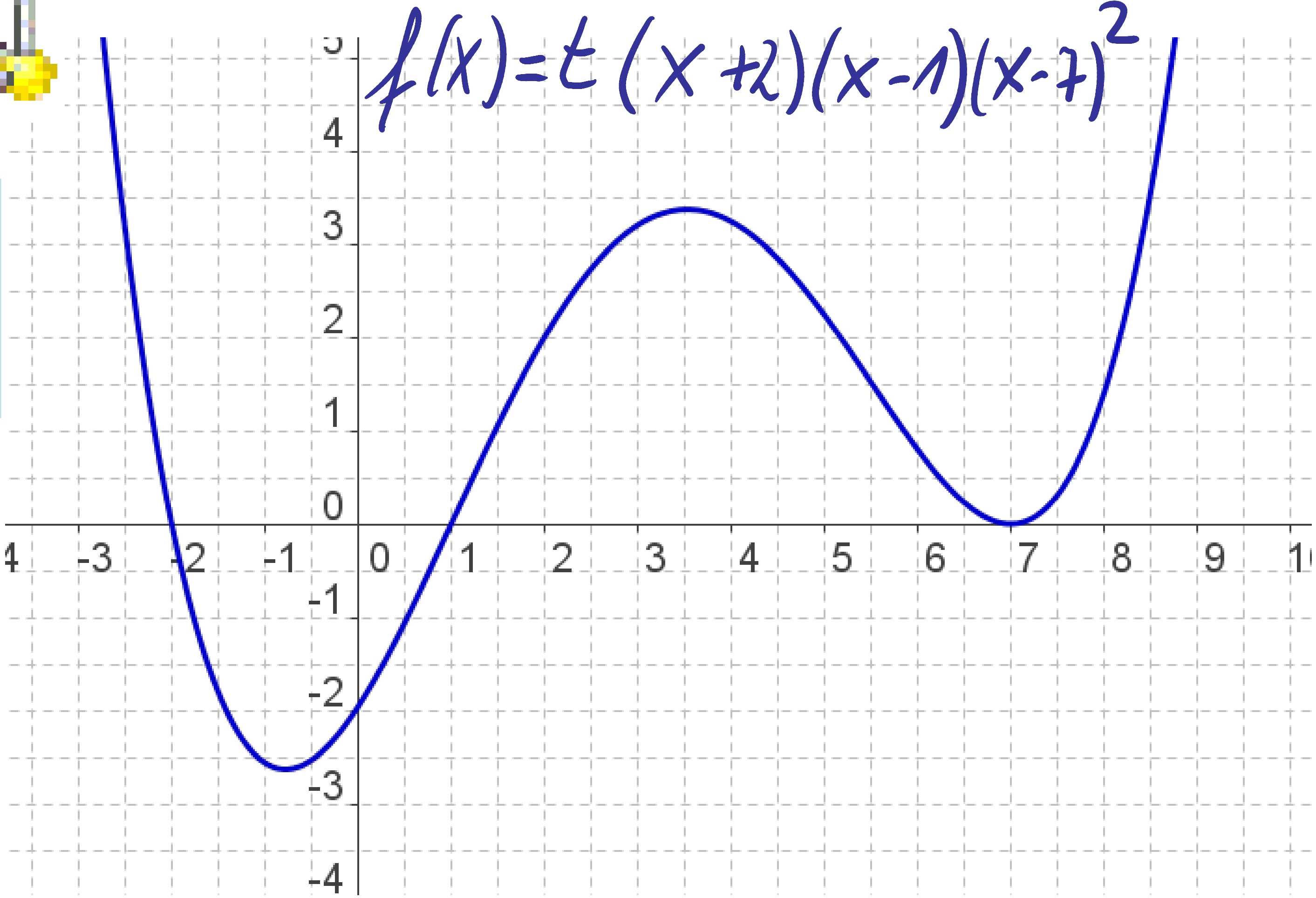


Which Equation ist Possible for this Polynomial?



Welche Gleichung kann dieses Polynom haben?





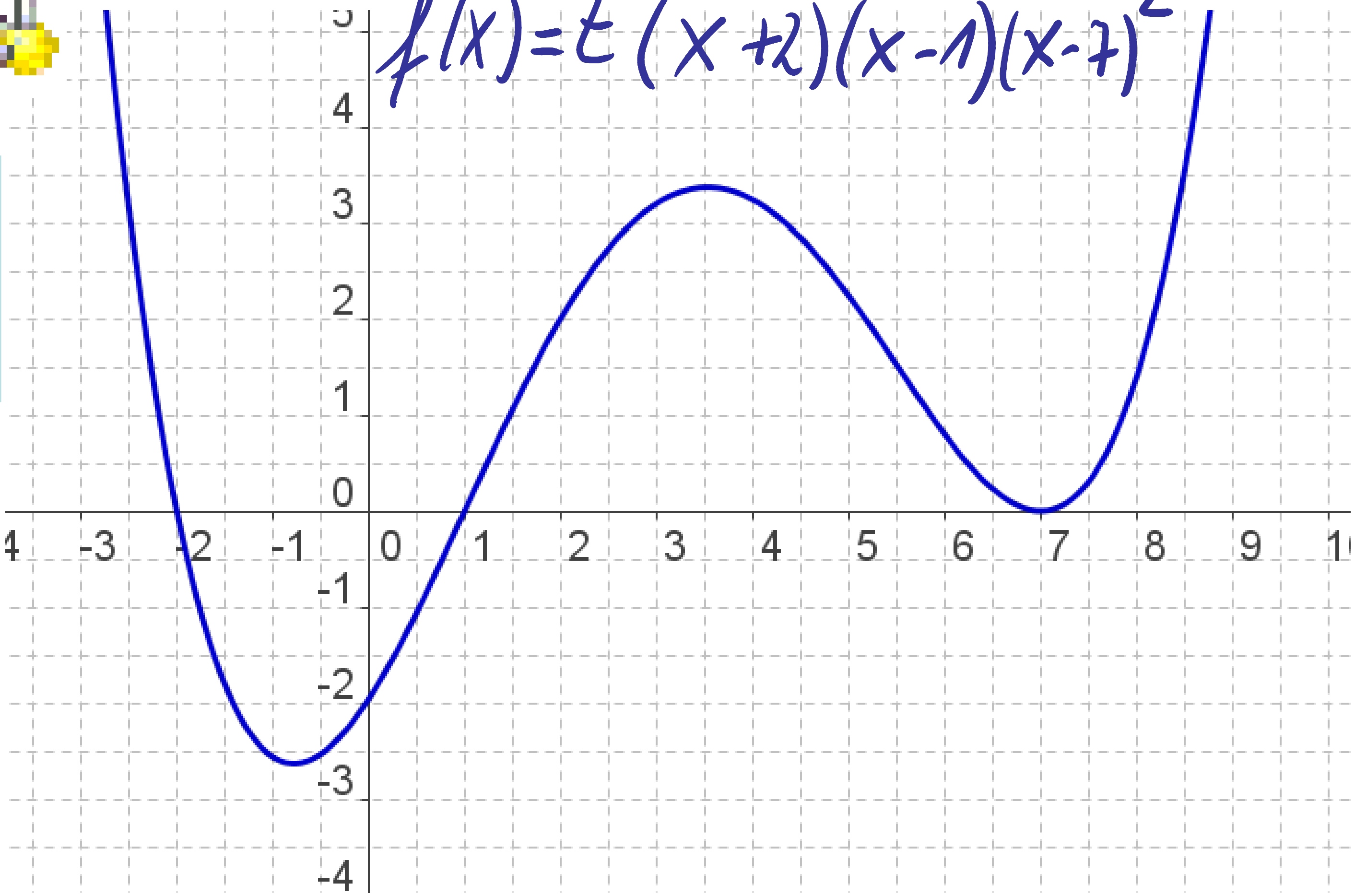
Diese Funktion

Vieta

Vieta, mehr

Which Equation ist Possible for this Polynomial?

$$f(x) = t\,(x+2)(x-1)(x-7)^2$$



this

function

Vieta

Vieta, plus

Was ist eigentlich ein Polynom?

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

Ein Polynom ist eine Summe von Potenzfunktionen.

Der höchste Exponent, der vorkommt, heißt Grad des Polynoms.

- Polynome 1. Grades sind die Geraden
- Polynome 2. Grades sind die Parabeln
- Polynome 3. Grades haben immer eine symmetrische s-Form.
- Polynome 4. Grades haben höchstens 3 Extrema.
- Je höher der Grad, desto vielfältigere Formen sind möglich.

How is a Polynomial Defined?

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

A polynomial is a sum of power functions.

The highest exponent is named **degree of the polynomial**.

- Polynomials of degree 1 are the straight lines.
- Polynomials of degree 2 are the parabolas.
- Polynomials of degree always have a symmetrical s-Form.
- Polynomials of degree 4 have at most 3 extrema.
- The higher the degree the manifold forms are possible.

Polynome und ihre Linearfaktoren

Jede reelle Nullstelle a
erzeugt einen Linearfaktor. $(x-a)$

$$f(x) = (x-a)\,q(x)$$

Wenn das Restpolynom auch noch die Nullstelle a
enthält, kann man den Linearfaktor mehrfach „herausziehen“.

$$f(x) = (x-a)^k\,p(x) \quad mit \quad p(a) \neq 0$$

Geht das maximal k-mal, dann heißt a k-fache Nullstelle,
oder „Nullstelle der Vielfachheit k“

Polynomials and their Linear Factors

Every real zero a
corresponds to a linear factor. $(x-a)$

$$f(x) = (x-a)\, q(x)$$

If the remaining polynomial has zero a too,

you can „pull out“ the linear factor twice ore more.

$$f(x) = (x-a)^k\ p(x) \quad mit \quad p(a) \neq 0$$

a is named **zero of degree k**, if this is possible k times at most,
An othe name ist „**zero of the order k**“.

Polynome und ihre Linearfaktoren

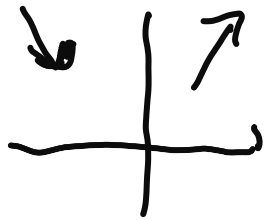
$$f(x) = (x-a)^k \, p(x) \quad mit \quad p(a) \neq 0$$

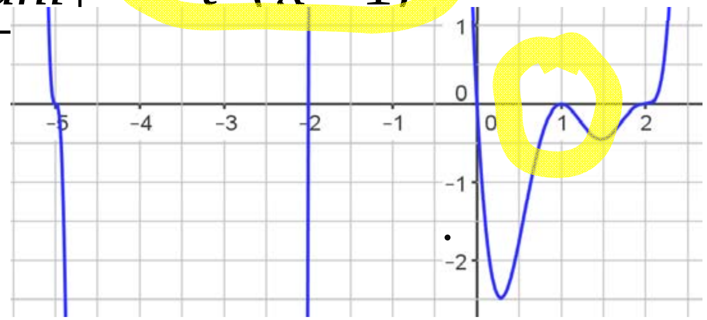
In der Nähe eine k-fachen Nullstelle verhält sich das Polynom wie sich die k-Potenzfunktion im Ursprung verhält.

$$f(x) = (x+5)^3 (x+2) \; x \; (x-1)^2 \, (x-2)^3$$

$$f(nahe\ 1) = \boxed{pos.Zahl} \bullet (x-1)^2 \bullet \boxed{neg.Zahl} \approx -t \cdot (x-1)^2$$

Grad 10 Gesamtverlauf





Ein Polynom n-ten Grades hat höchsten n Nullstellen, mit ihrer Vielfachheit gezählt. Fundamentalsatz der Algebra (reell)

Polynomials and their Linear Factors

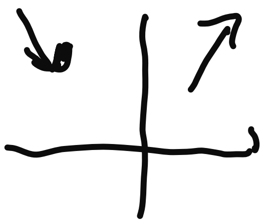
$$f(x) = (x-a)^k \; p(x) \quad mit \quad p(a) \neq 0$$

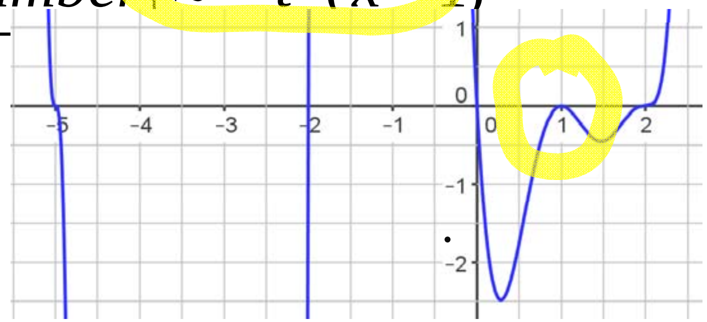
The polynomial is in the neighbourhood of k-order zero similar to a power function of degree k near the origin.

$$f(x) = (x+5)^3 (x+2) \; x \; (x-1)^2 \, (x-2)^3$$

$$f(near\ 1) = \boxed{pos.number} \bullet (x-1)^2 \bullet \boxed{neg.number} \approx -t \cdot (x-1)^2$$

degree 10 outside form





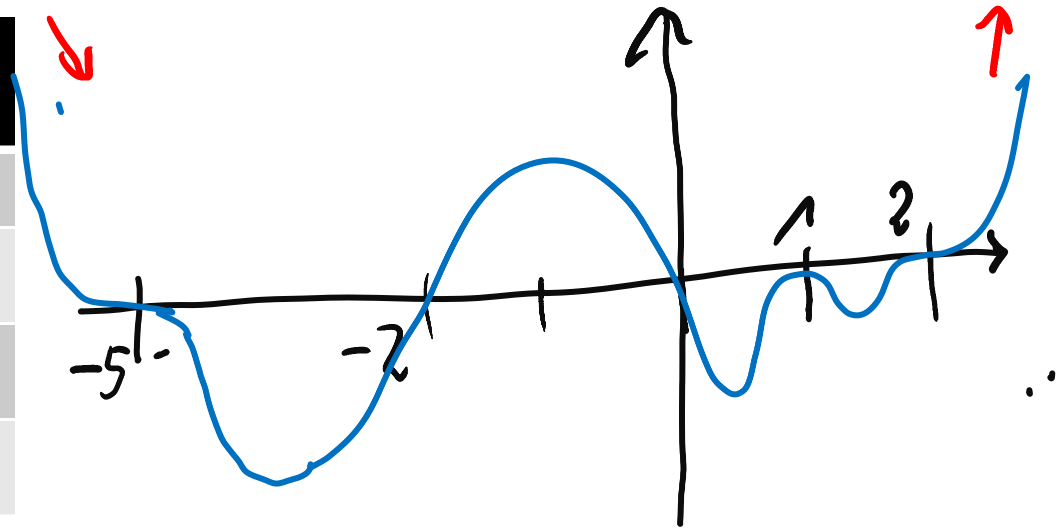
A polynomial of degree n has at most n zeros, counted with their orders. fundamental theorem of algebra (real version)

Polynome und ihre Linearfaktoren

$$f(x) = (x+5)^3 (x+2)\ x\ (x-1)^2 (x-2)^3$$

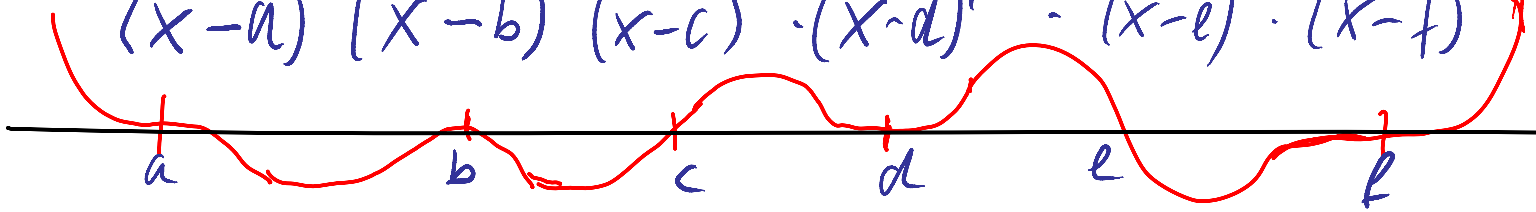
Qualitativer Graph eines durch Linearfaktoren gegeben Polynoms

Vorzei-chen	Grad	Ges. Verlauf
+	gerade	↘ ↗
−	gerade	↗ ↘
+	ungerade	↗ ↗
−	ungerade	↘ ↘



$$(x-a)^3 (x-b)^2 (x-c) \cdot (x-d)^4 \cdot (x-e) \cdot (x-f)^5$$

Grad?



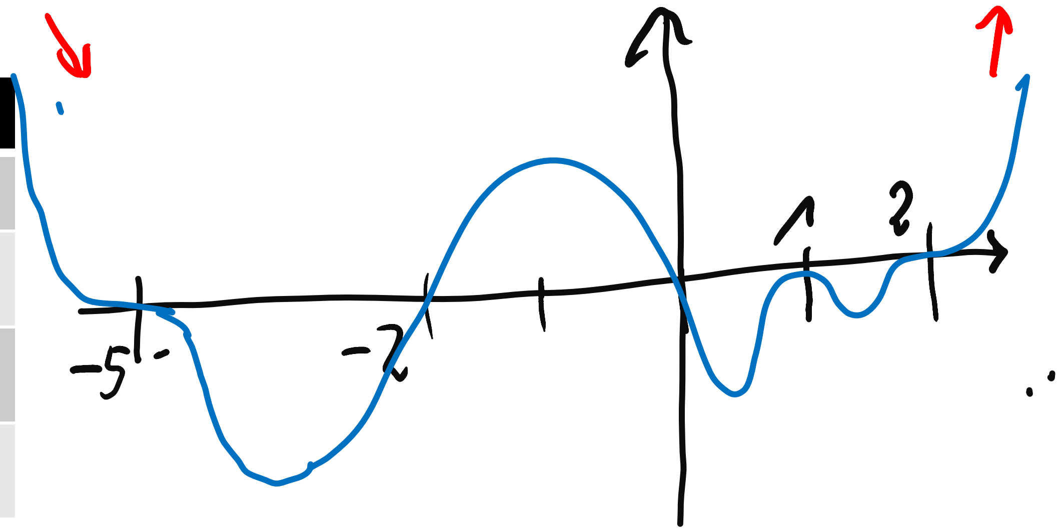
Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2013 http://www.leuphana.de/matheomnibus

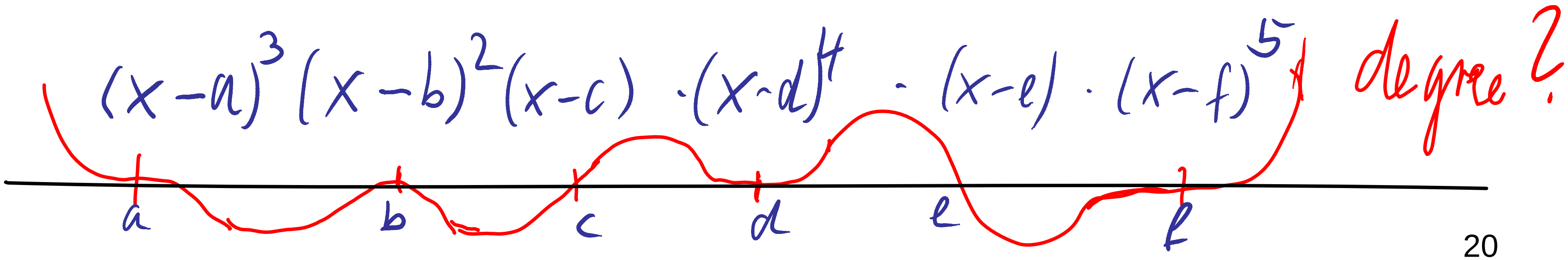
Polynomials and their Linear Factors

$$f(x) = (x+5)^3 (x+2)\ x\ (x-1)^2\ (x-2)^3$$

Qualitative Graph of a polynomial, given with its linear factors.

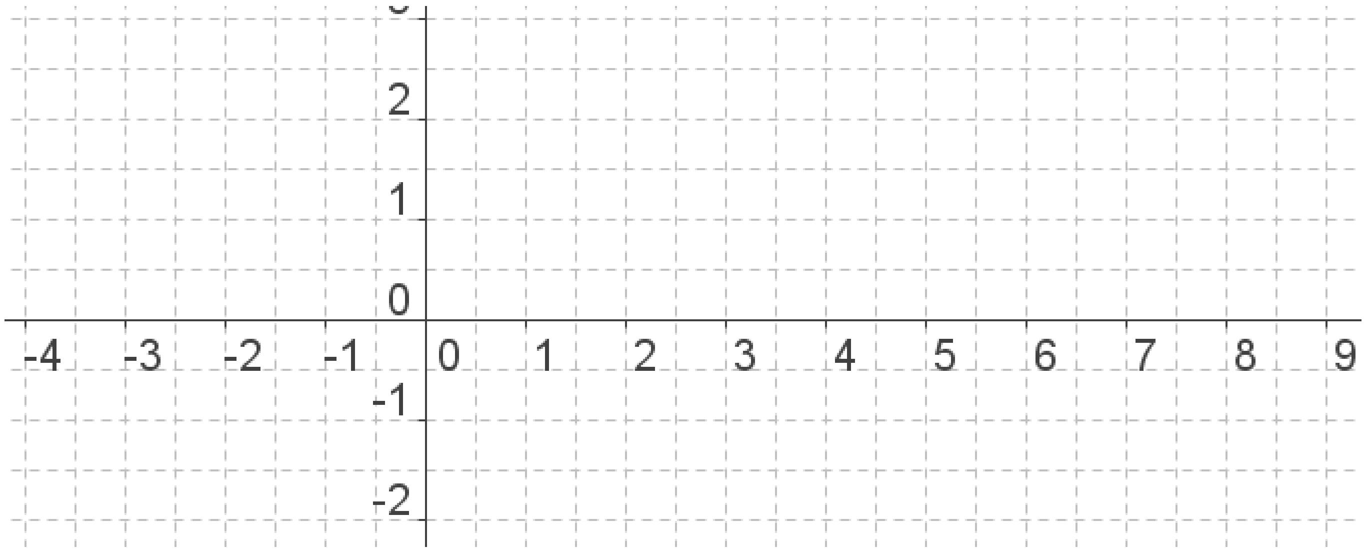
sign	degree	outer form
	even	
+	even	↘ ↗
−	odd	↗ ↘
+	odd	↗ ↗
−		↘ ↘





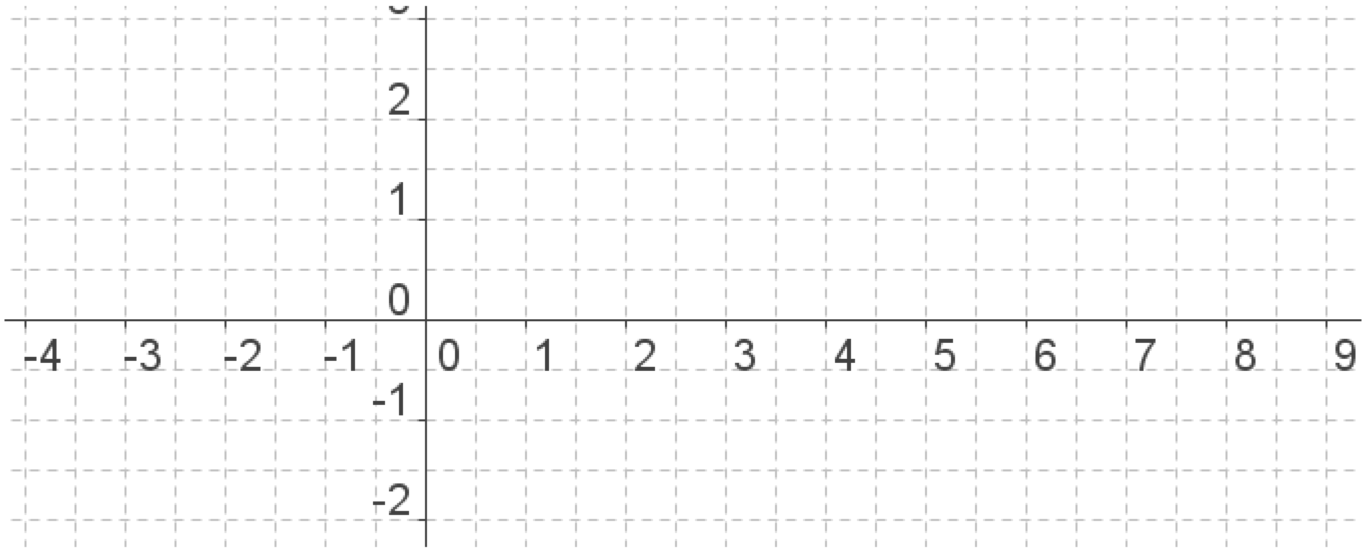
Übung 2 mit Polynomen

$$f(x) = (x+2)^2 \cdot (x-1)(x-7)^2$$



Practice 2 with Polynomials

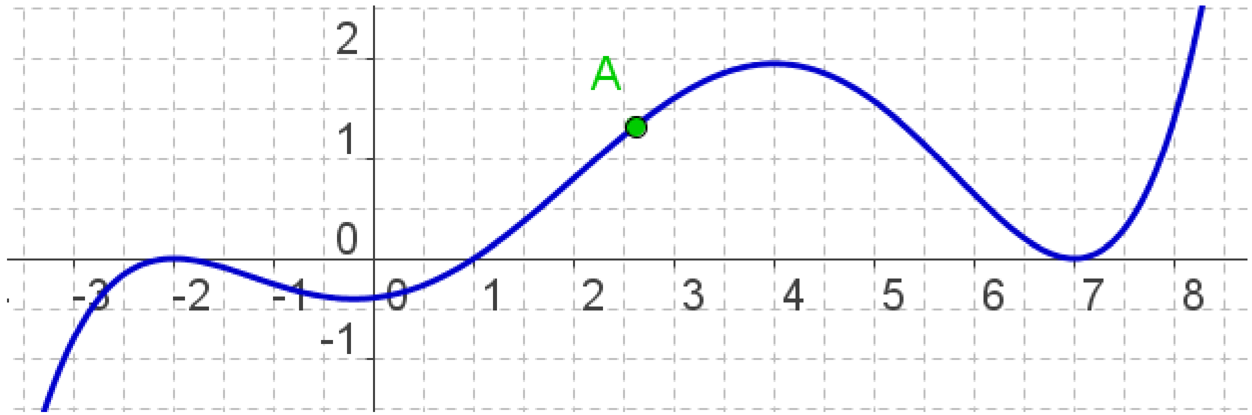
$$f(x) = (x+2)^2 \cdot (x-1)(x-7)^2$$



Übung 2 mit Polynomen

$f(x) = (x+2)^2 \cdot (x-1)(x-7)^2$

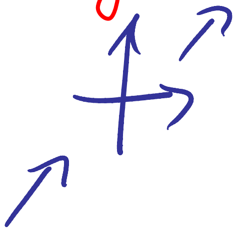
Grad5

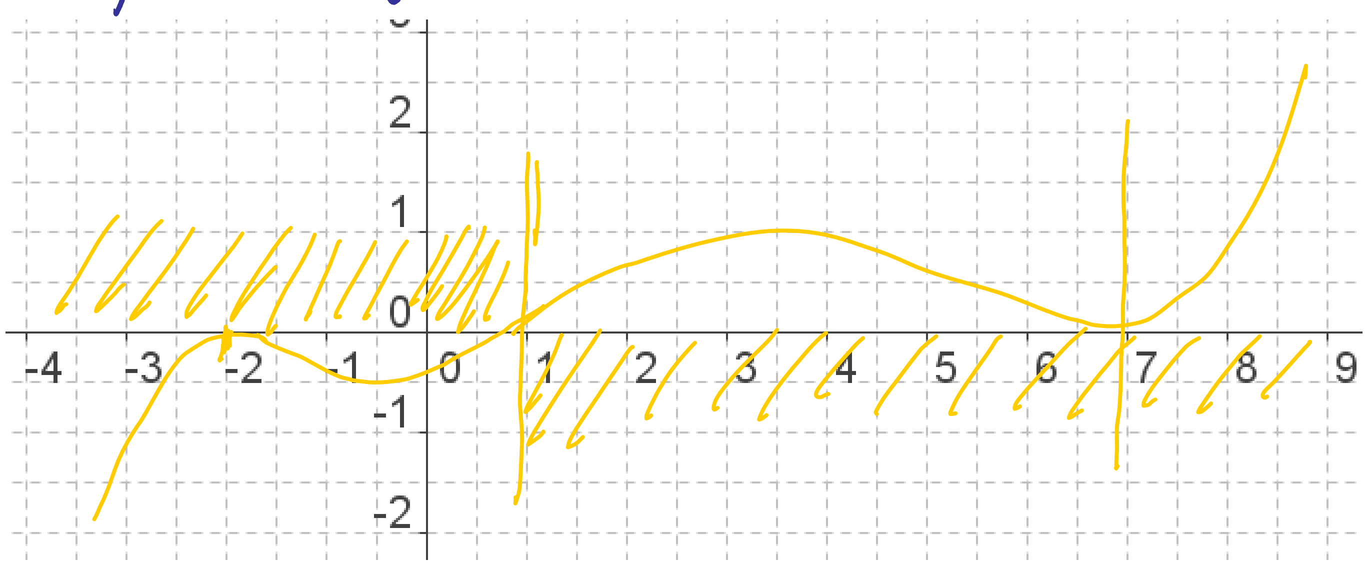


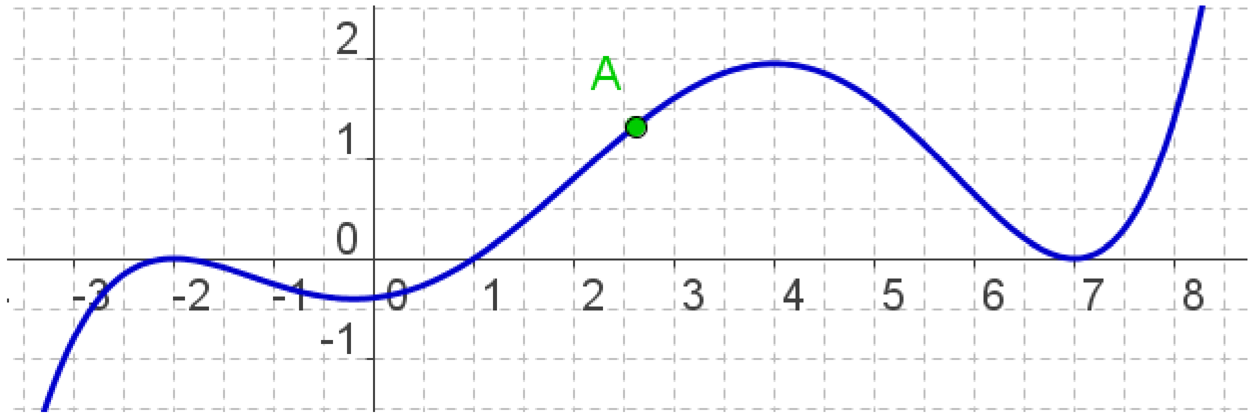
Practice 2 with Polynomials

$$f(x) = (x+2)^2 \cdot (x-1)(x-7)^2$$

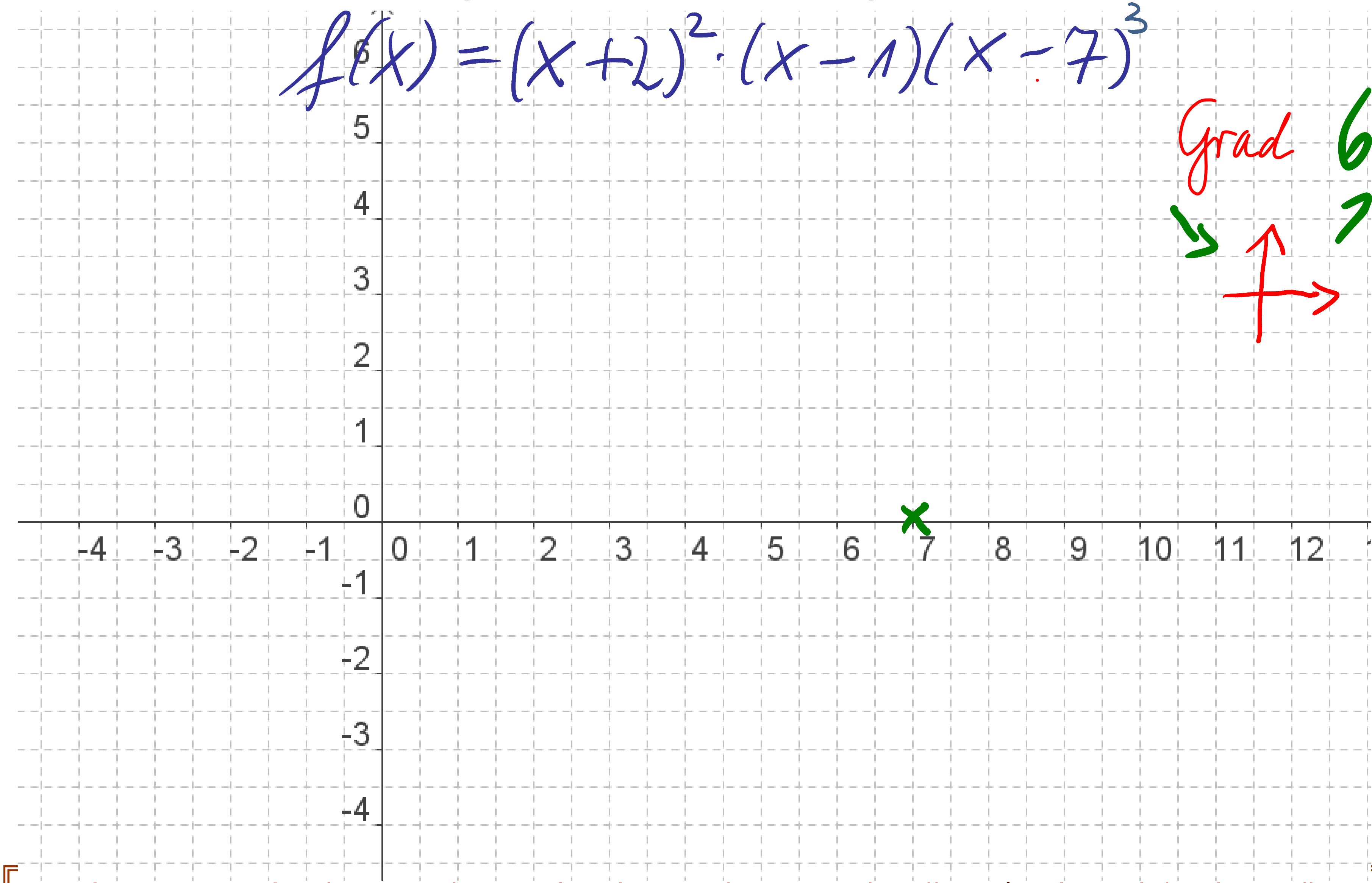
degree 5



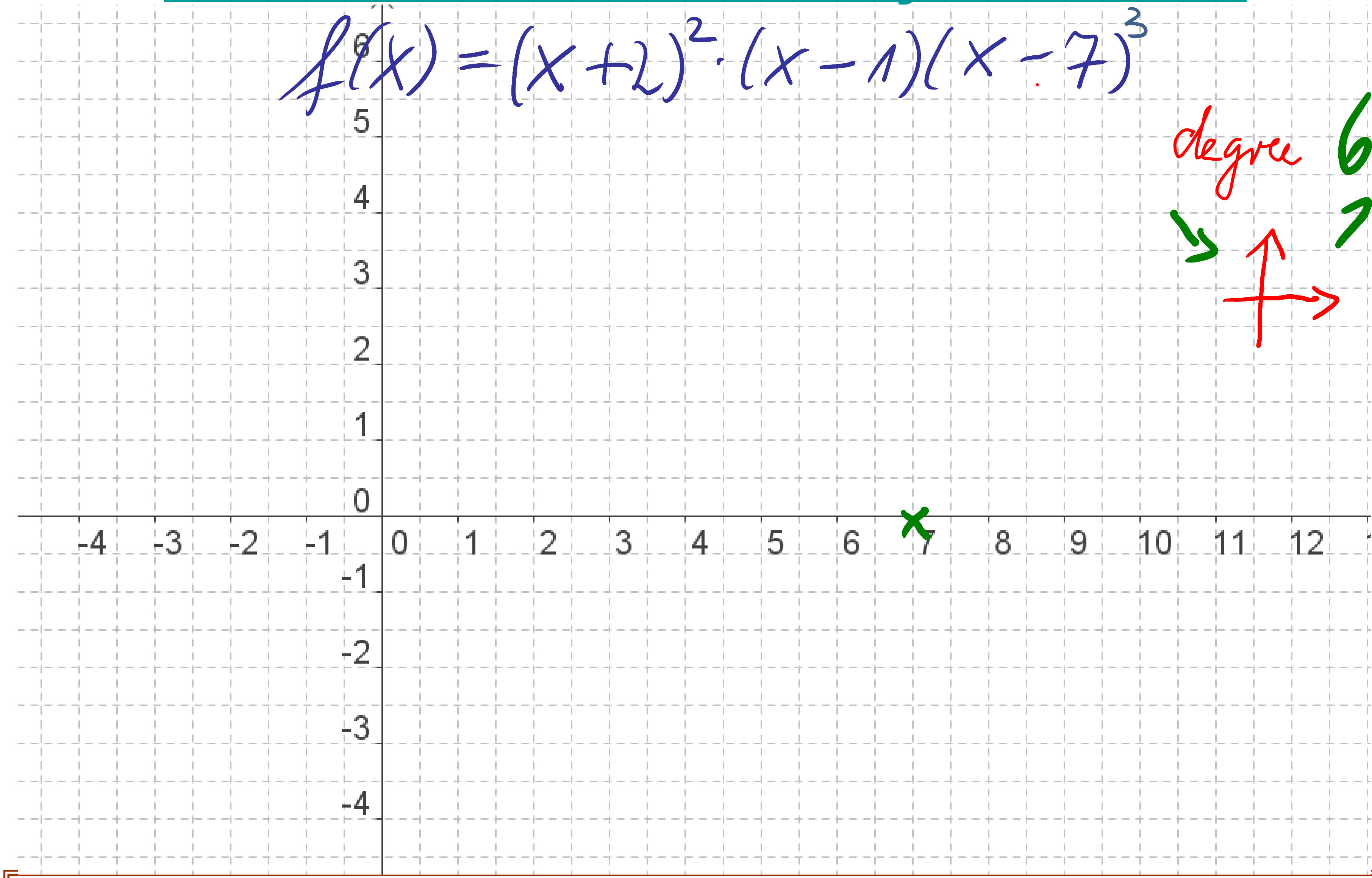




Übung 3 mit Polynomen

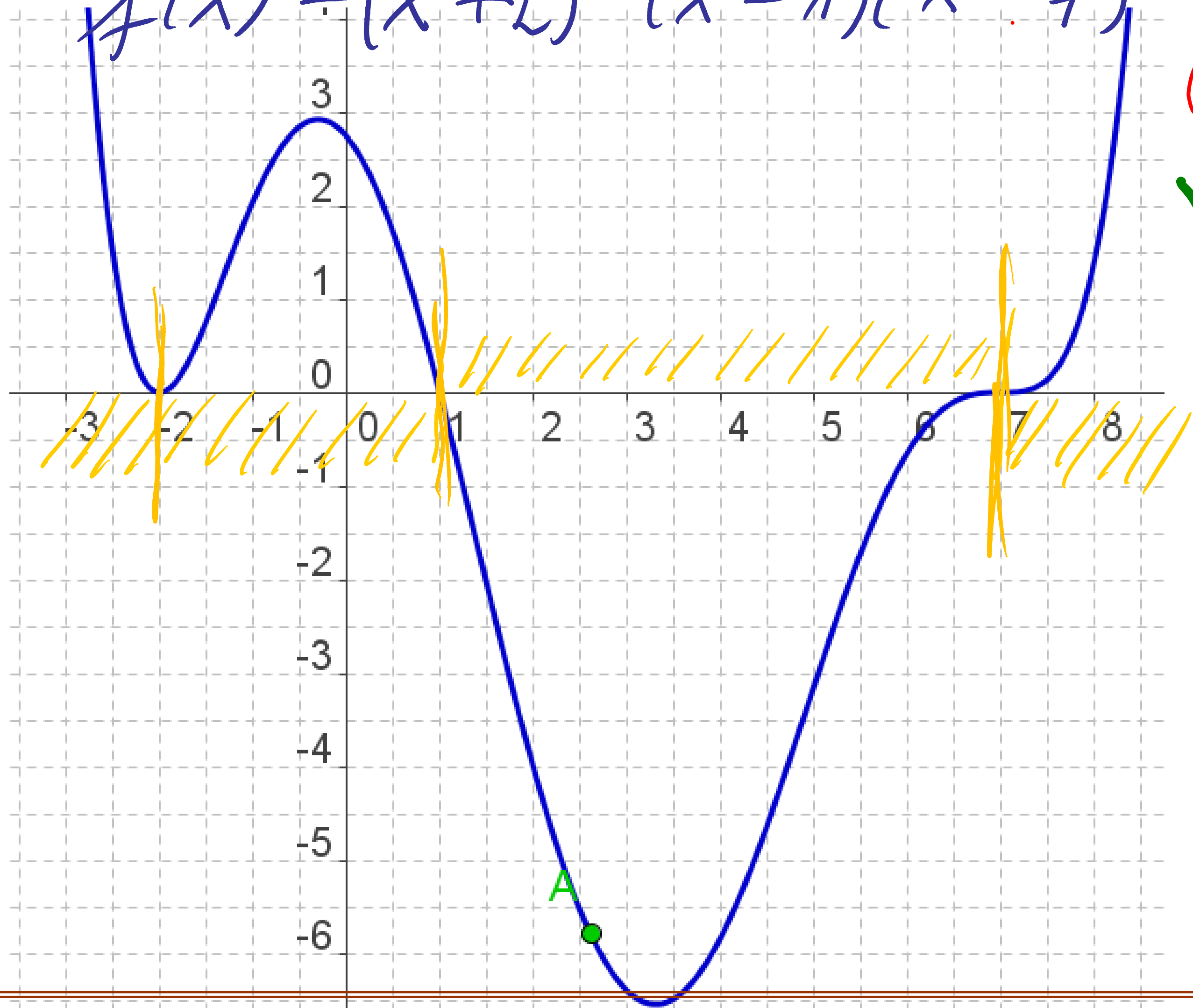


Practice 3 with Polynomials



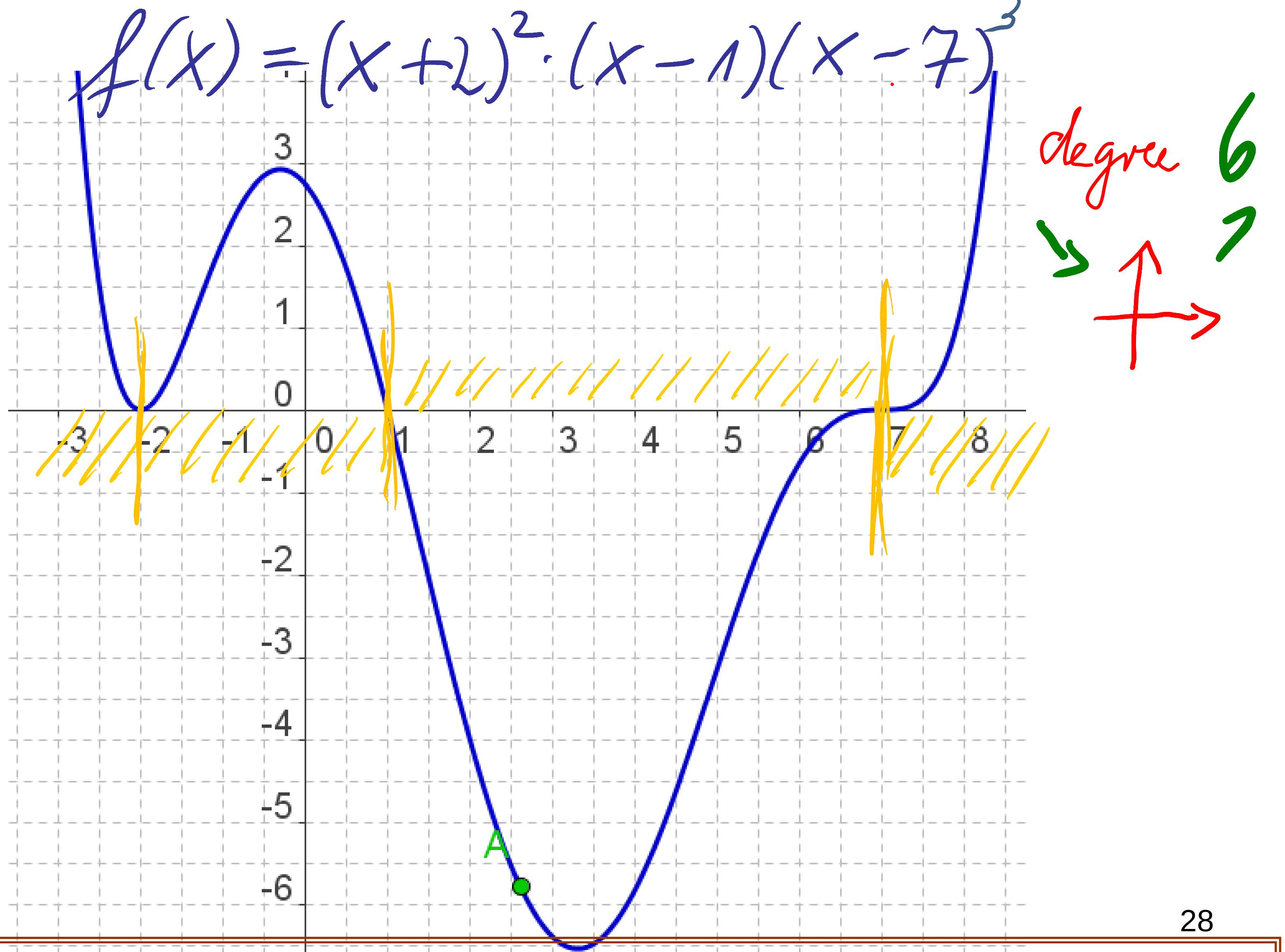
Übung 3 mit Polynomen

$f(x) = (x+2)^2 \cdot (x-1)(x-7)^3$

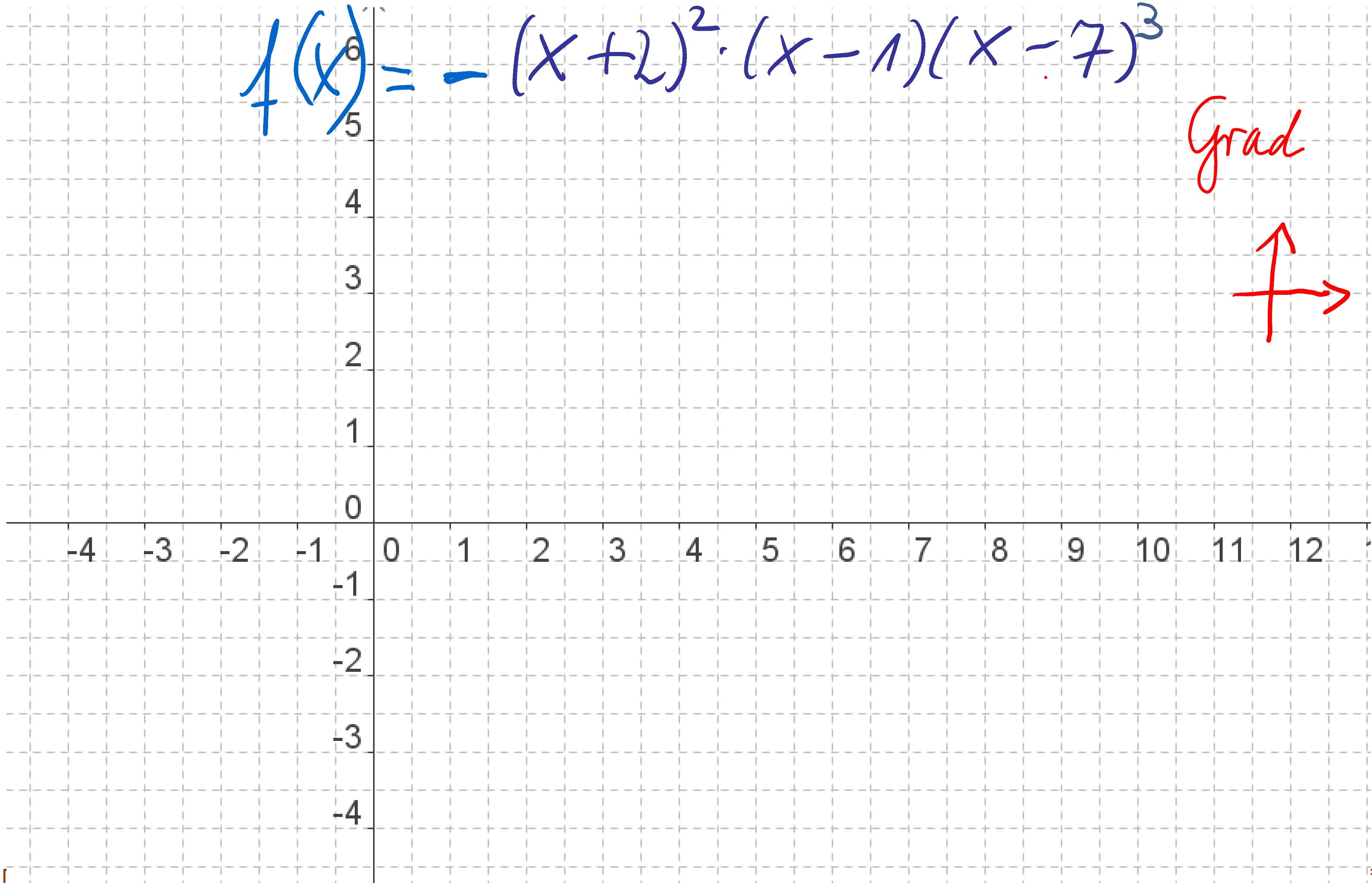


Grad 6

Practice 3 with Polynomials

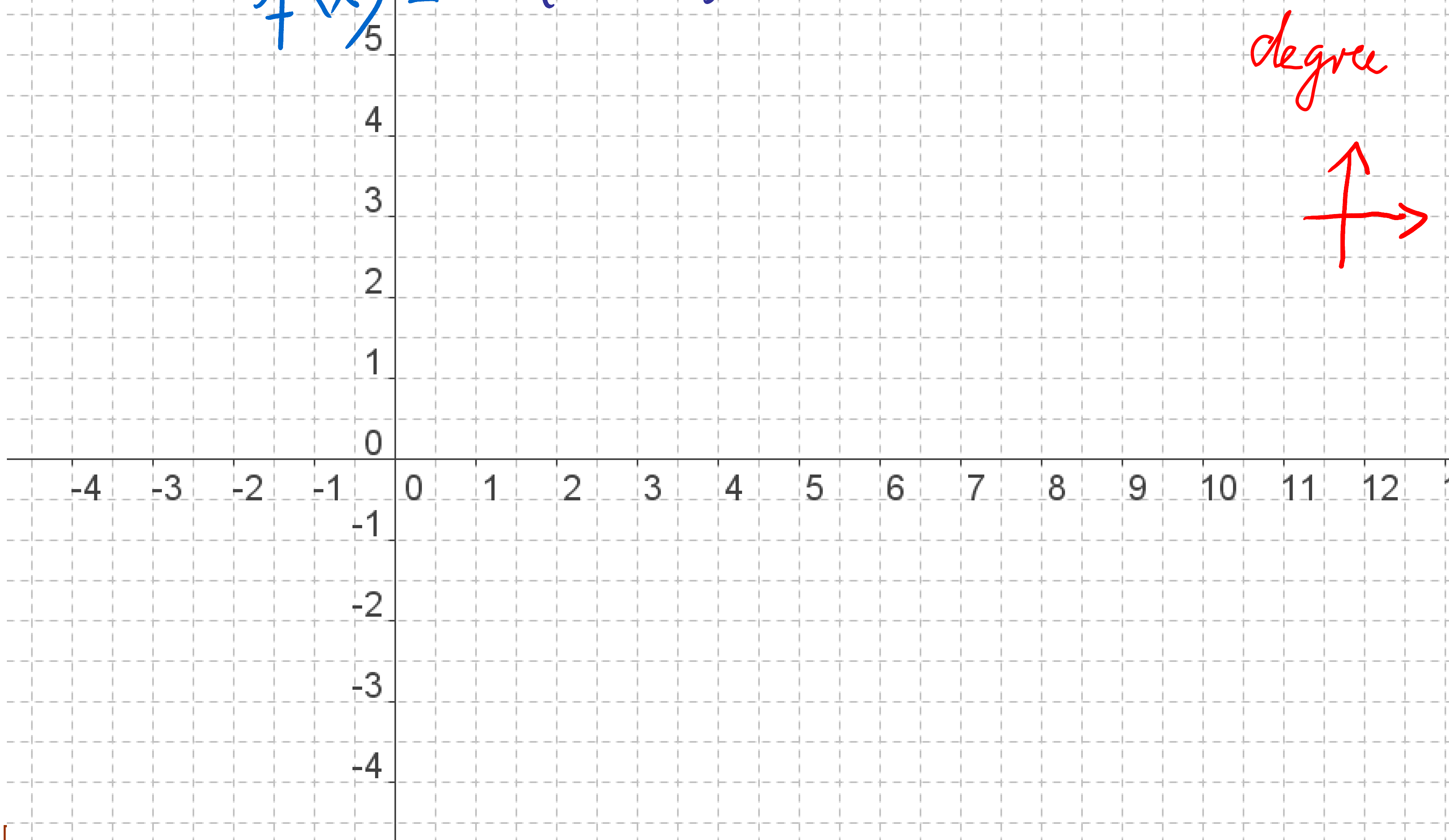


Übung 4 mit Polynomen

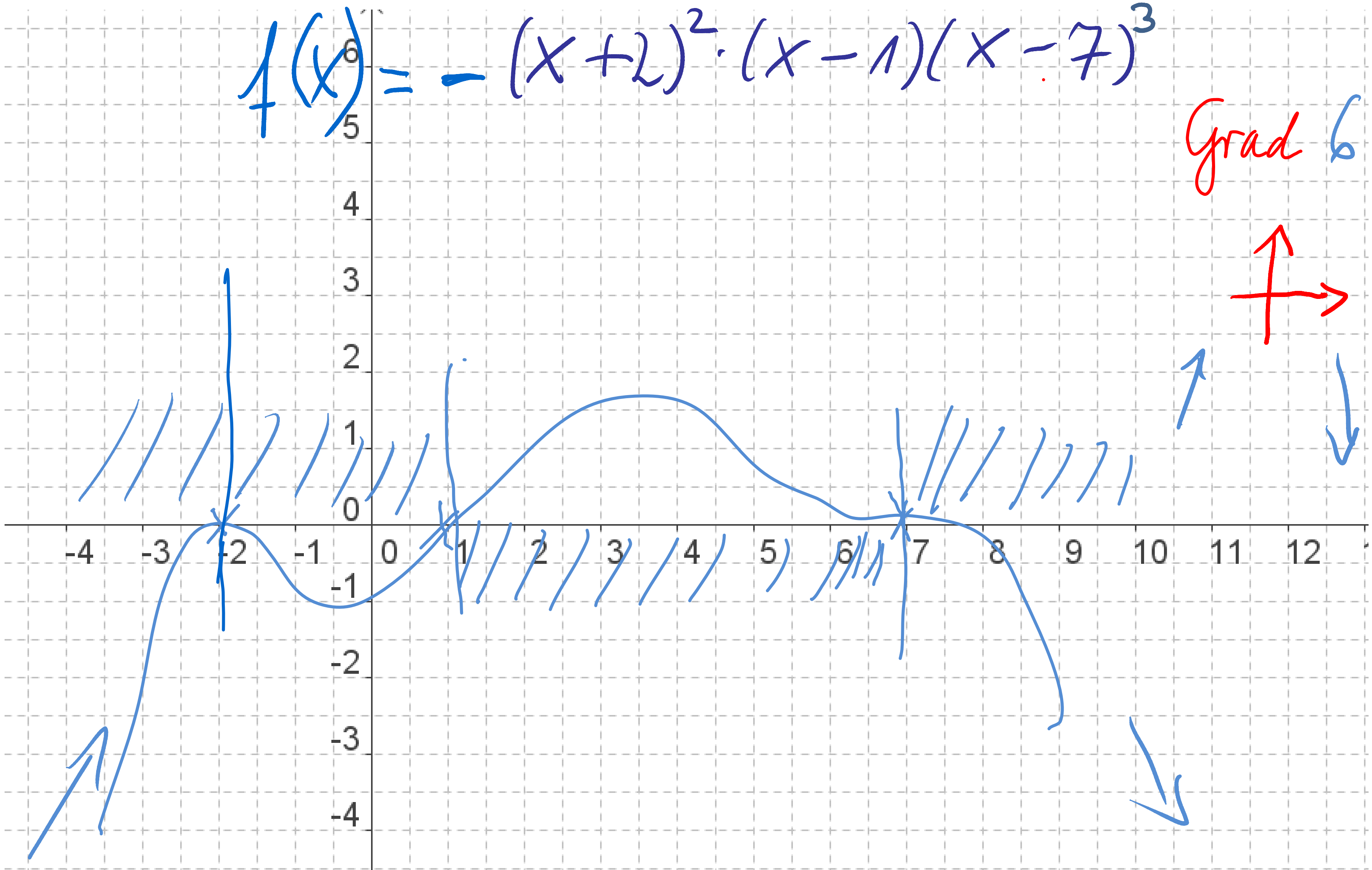


Practice 4 with Polynomials

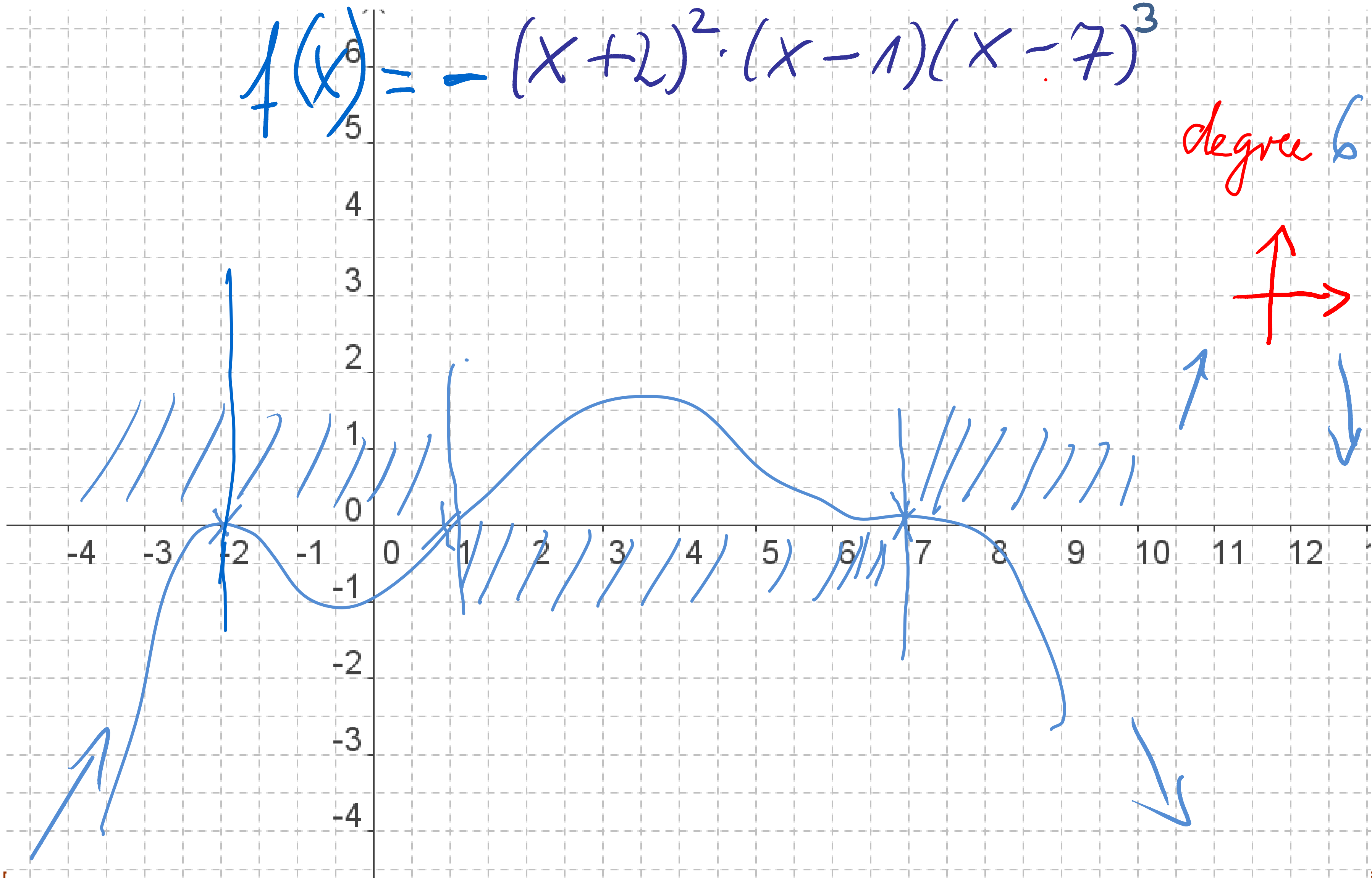
$$f(x) = -(x+2)^2 \cdot (x-1)(x-7)^3$$



Übung 4 mit Polynomen



Practice 4 with Polynomials



Funktionen als zentrales Werkzeug

- Potenzfunktionen
- Polynome
- **Trigonometrische Funktionen**
- **Exponentialfunktionen**
- **Davon so manche Umkehrfunktionen**
  - Wurzelfunktionen
  - Arkusfunktionen
  - **Logarithmusfunktionen**

Das war‘s dann aber auch

Und das noch koppeln mit $+ \quad - \quad \cdot \quad / \quad \wedge$ und Verkettung.

Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2013 http://www.leuphana.de/matheomnibus

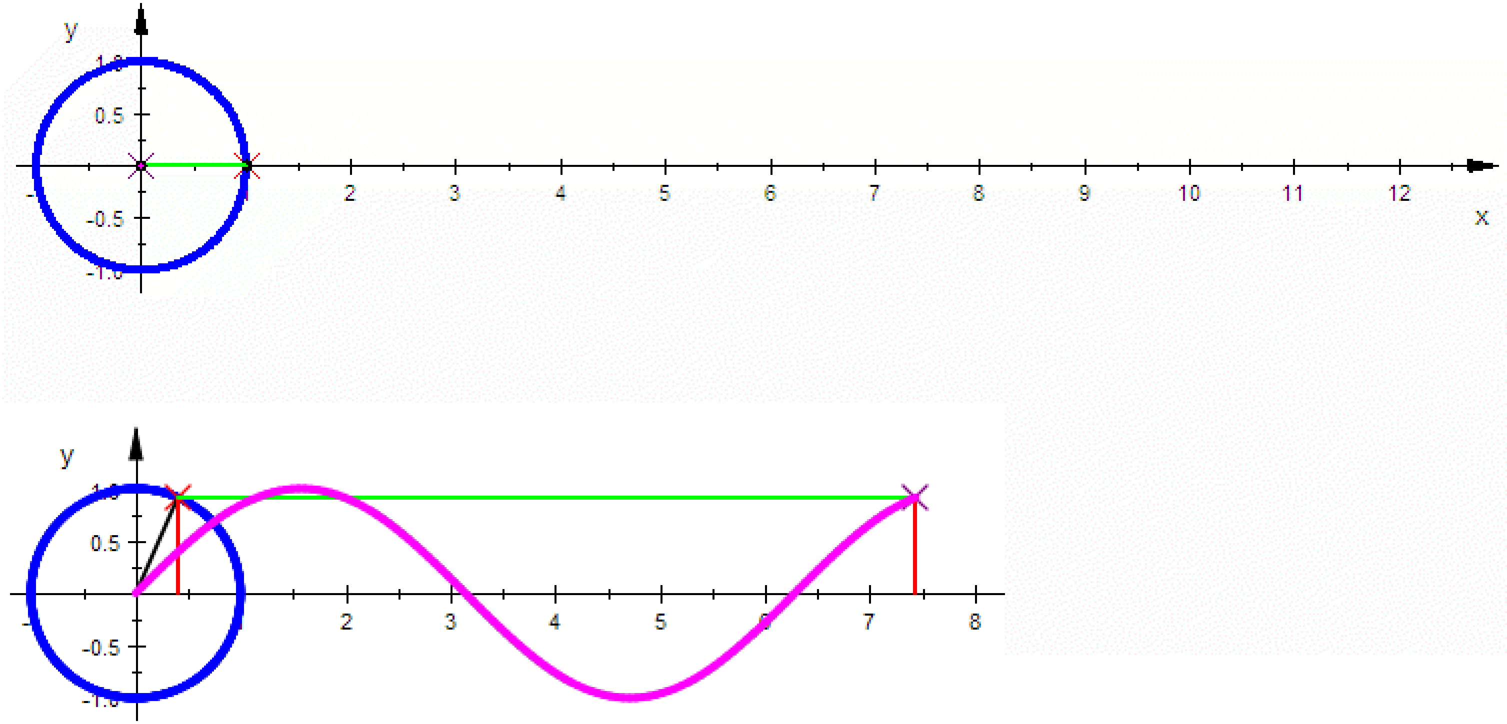
Functions as a Central Tool

- power functions
- polynomials
- **trigonometric functions**
- **exponential functions**
- **some inverse functions**
  - root functions
  - arc functions
  - **logarithmic functions**

that‘s all for normal purpose

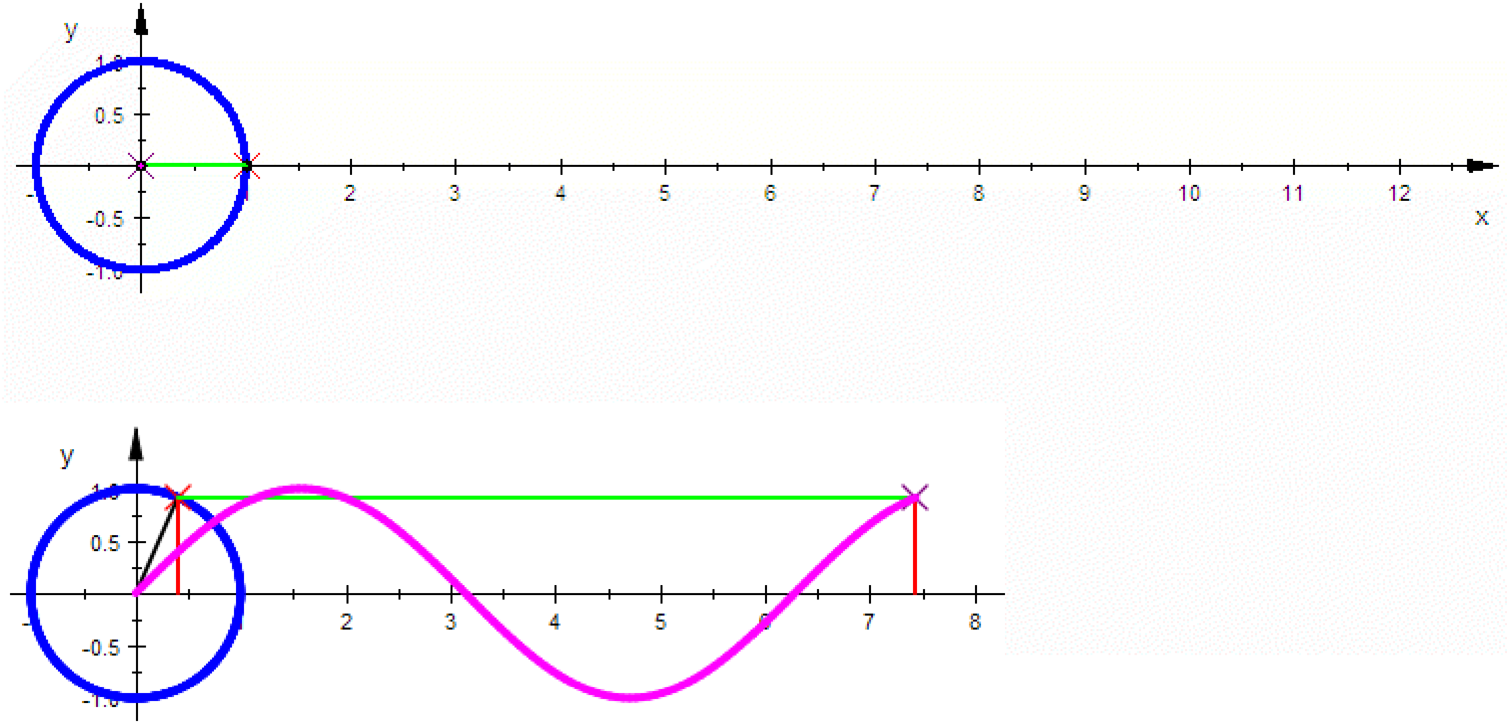
and at that you can caculate + − . • / ^ and build chains.

Funktionen als zentrales Werkzeug



Sinusfunktion

Functions as a Central Tool



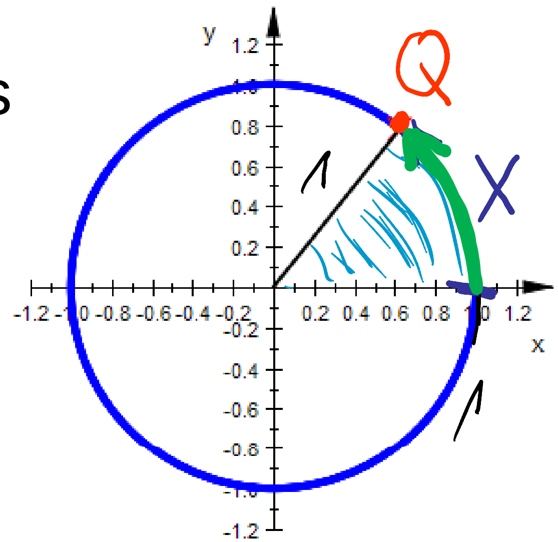
sine function

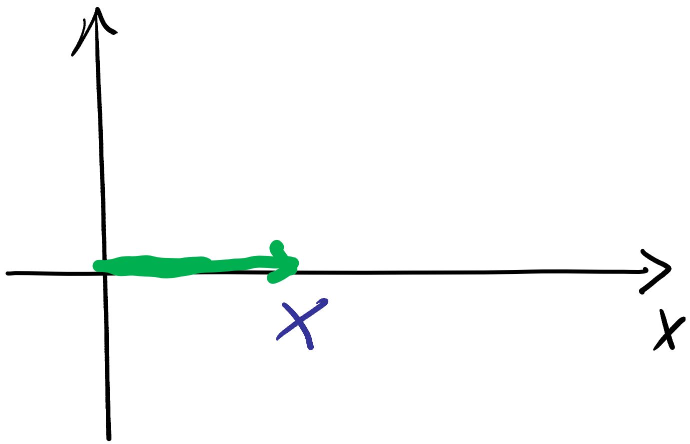
Die Winkel-Funktionen

Der Punkt Q läuft im Einheitskreis vom Start (1/0).
(mathematisch positiv = gegen die Uhr)

Den von Q zurückgelegten Weg x nennt man auch
„das Bogenmaß des Winkels“, um den sich Q gedreht hat.
Kurz: x ist der Winkel im Bogenmaß

Einheitskreis





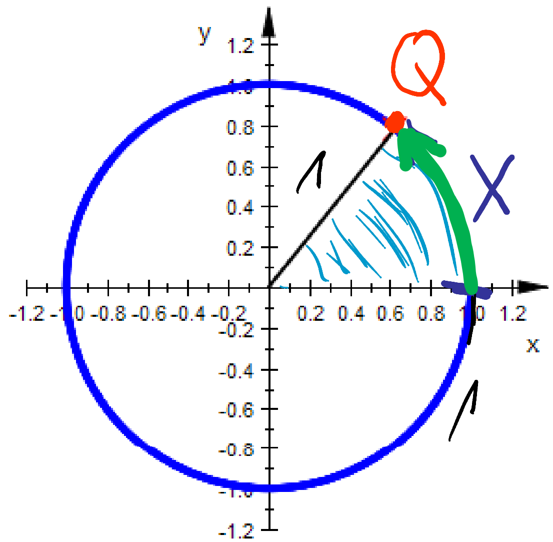
x wird Argument einer Funktion

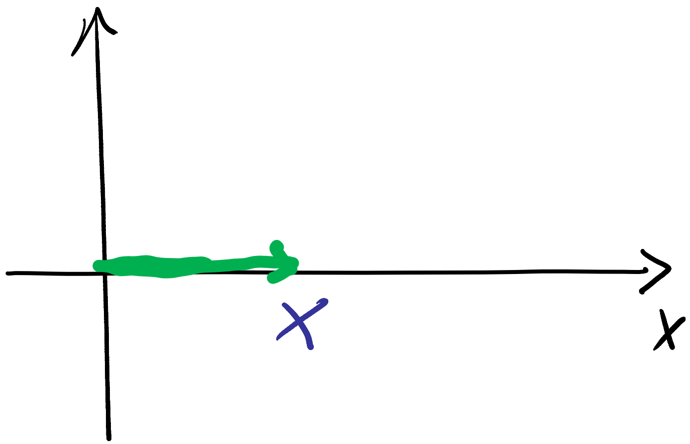
Angle Functions

Point Q goes in the unit circle with start in (1/0).
(mathematical positiv = against the clock)

The distance x, covered by Q on the circle, ist named „the radian measure of the angle“, the angle of rotation of Q.
short: x is the angle in radian measure

unit circle





x becomes argument of a function

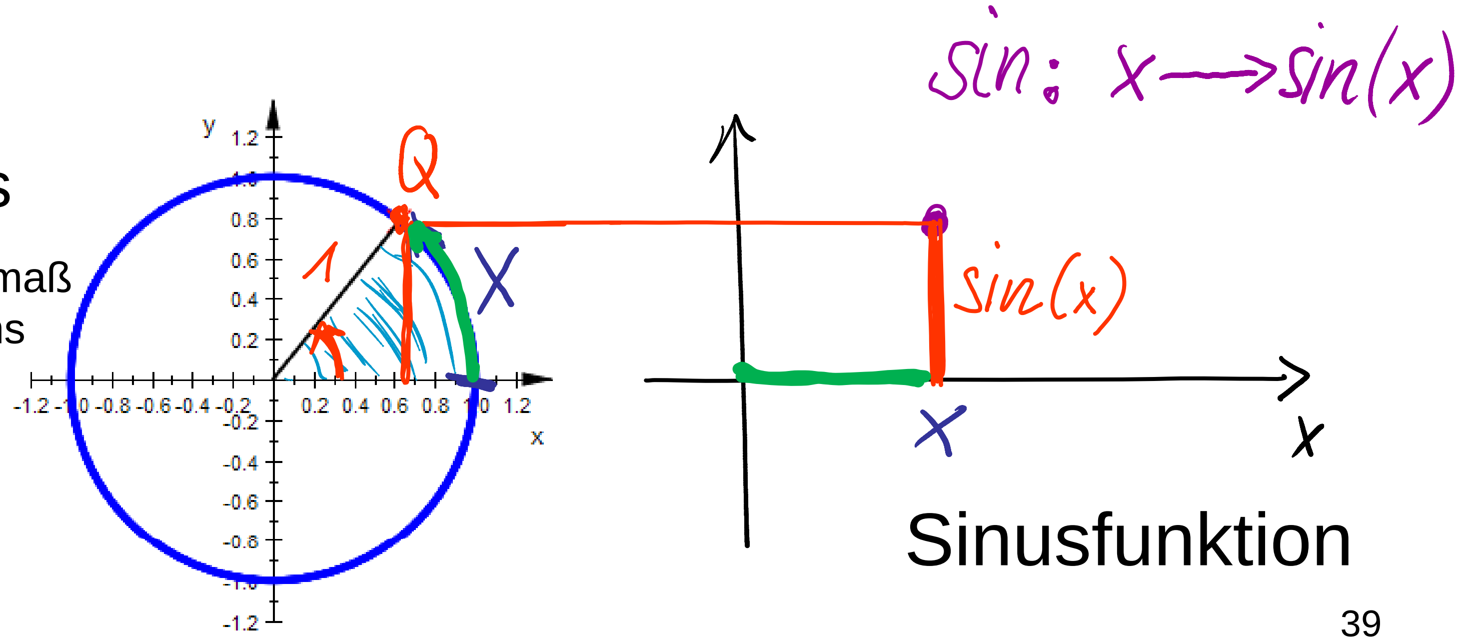
Die Sinus-Funktion

Dem Winkel x wird nun die Ordinate von Q zugeordnet.

Die Funktion, die das leistet, heißt Sinus-Funktion.

Einheitskreis

x = Winkel im Bogenmaß
= Länge des Bogens
im Einheitskreis



Sine Function

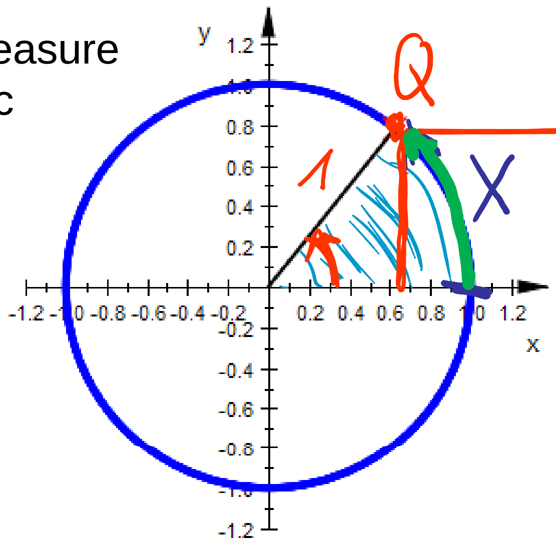
The angle x correlates with the ordinate of Q.

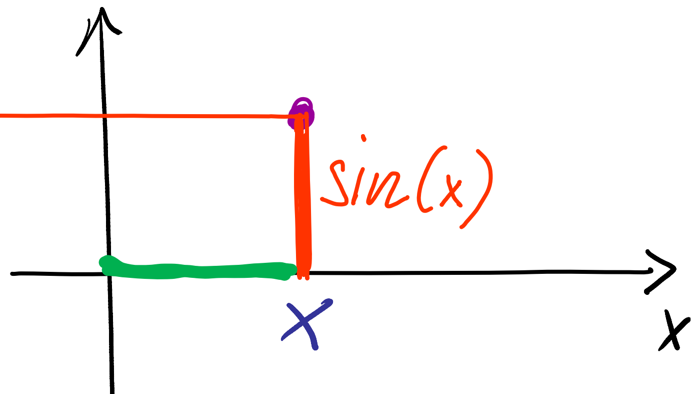
The function, which is able to do do, is the sine-function.

sin: x ⟶ sin(x)

unit circle

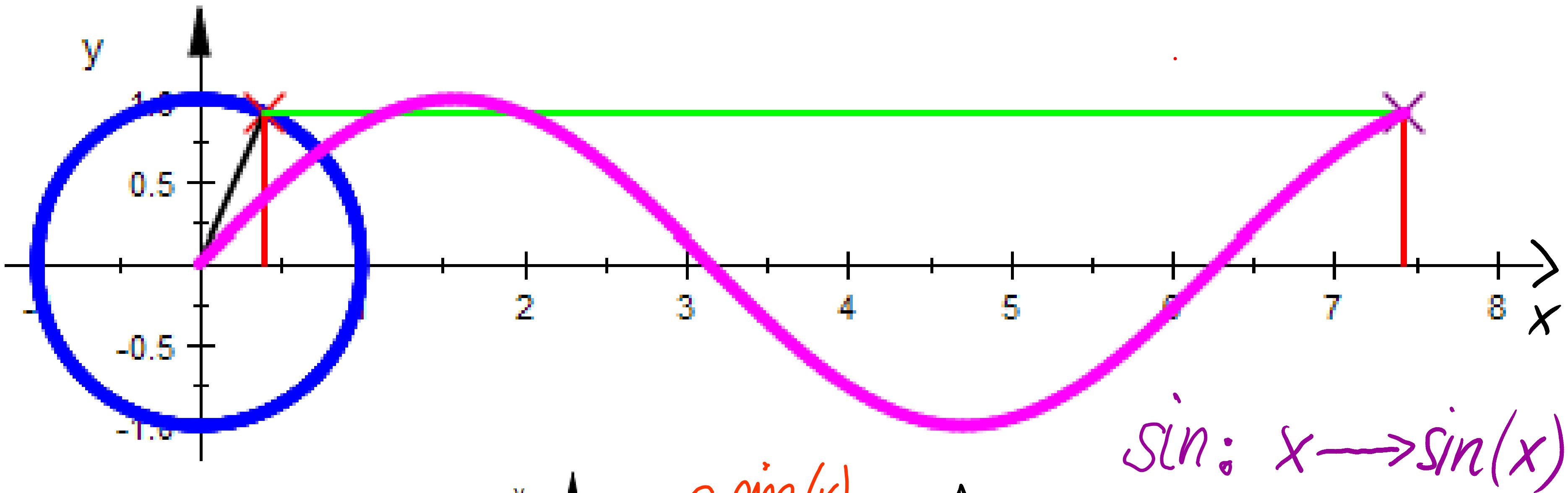
x = angle in radian measure
= distance of the arc
in the unit circle





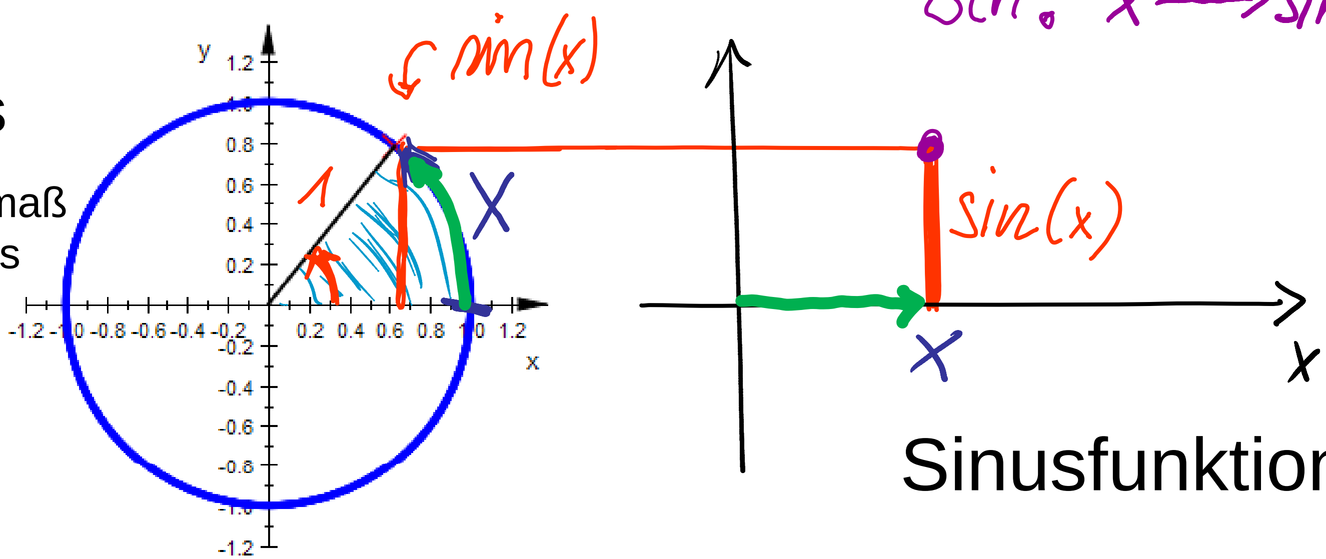
sine function

Die Sinus-Funktion



Einheitskreis

x = Winkel im Bogenmaß
= Länge des Bogens
im Einheitskreis



Sinusfunktion

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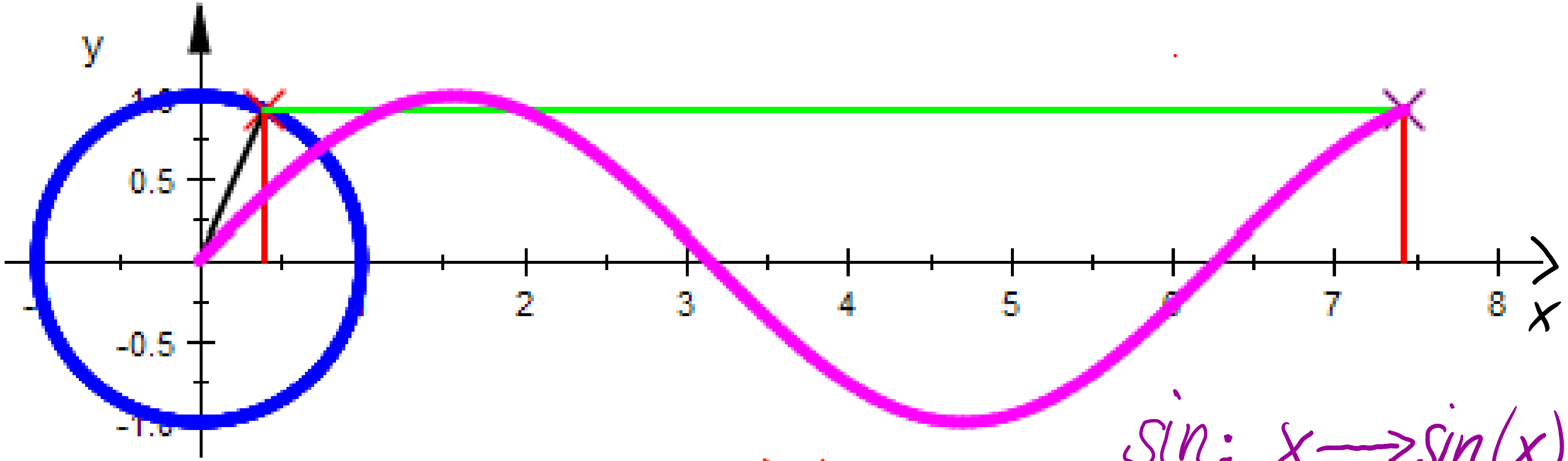
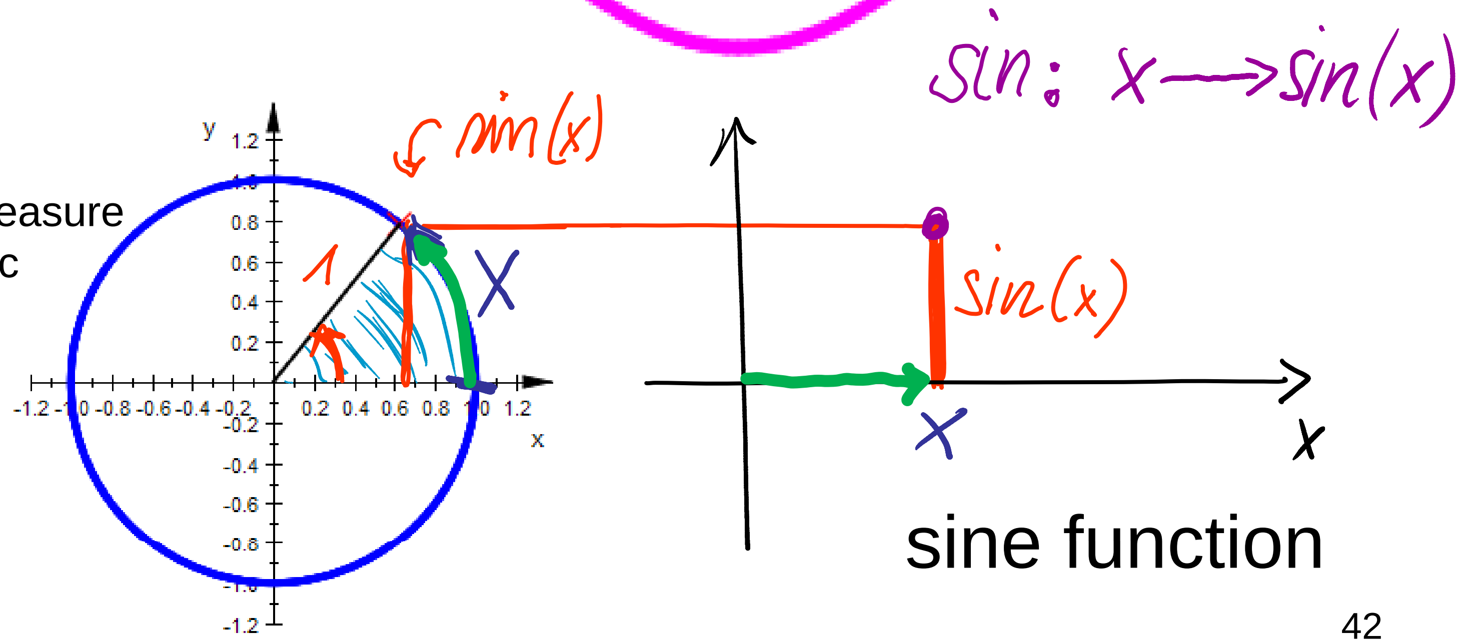
Sine Function



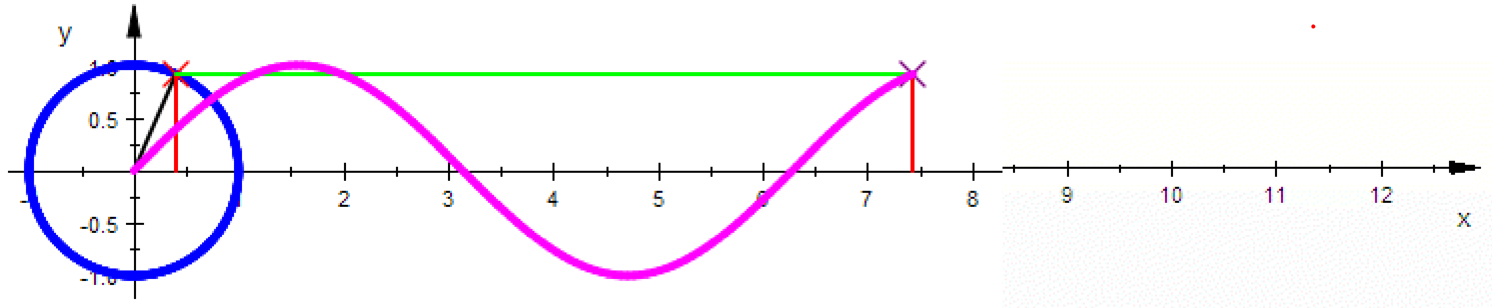
unit circle

x = angle in radian measure
= distance of the arc
in the unit circle

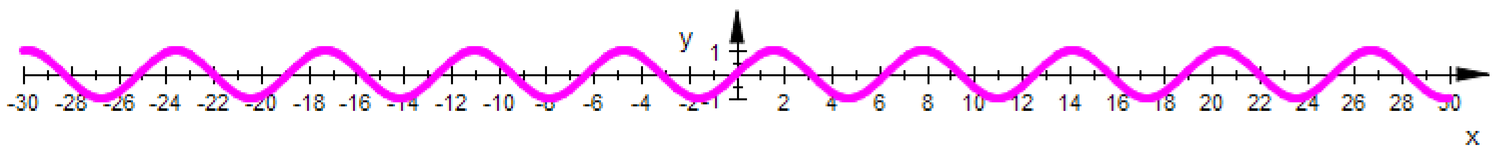


sine function

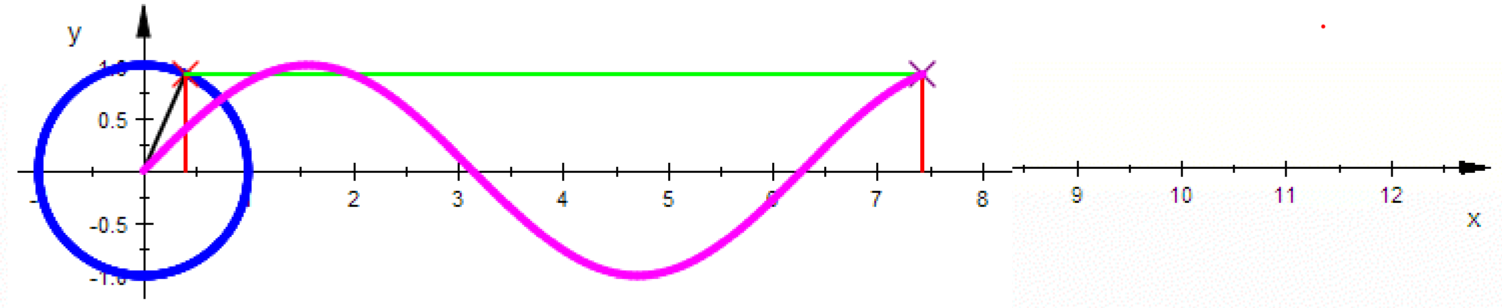
Eigenschaften der Sinus-Funktion



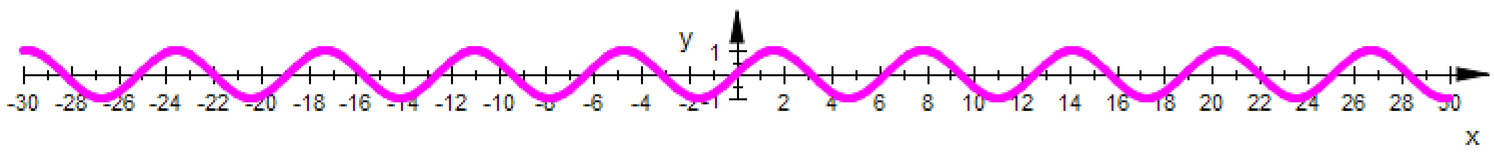
- Die Sinus-Funktion ist periodisch.
- Die Periode ist 2π .
- Die Sinuswerte liegen zwischen -1 und + 1.
- Die Sinusbögen sind symmetrisch.
- Die Sinuskurve ist punktsymmetrisch zum Ursprung



Properties of the Sine Function



- The sine function is periodic.
- The period is 2π .
- The sine ordinates are between -1 and + 1.
- The arcs of the sine function are symmetric.
- The sine curve has the origin as a point of symmetry.



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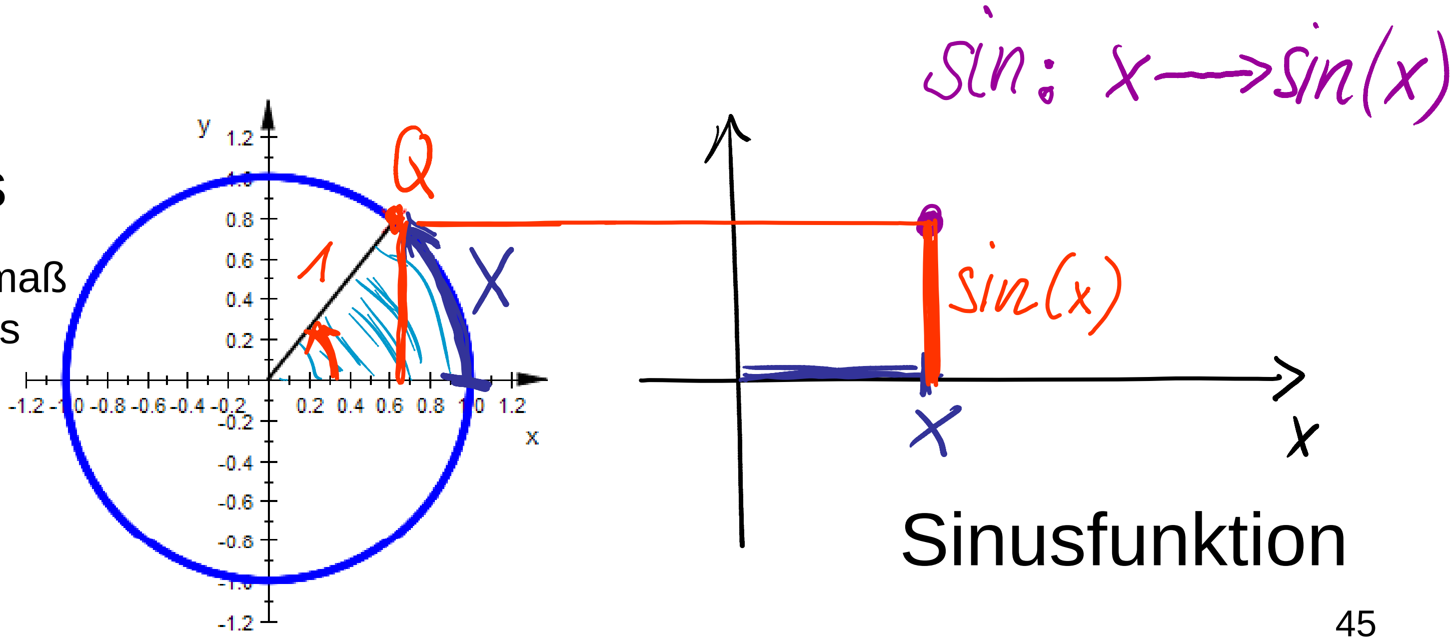
Die Sinus-Funktion

Dem Winkel x wird nun die Ordinate von Q zugeordnet.

Die Funktion, die das leistet, heißt Sinus-Funktion.

Einheitskreis

x = Winkel im Bogenmaß
= Länge des Bogens
im Einheitskreis



Sinusfunktion

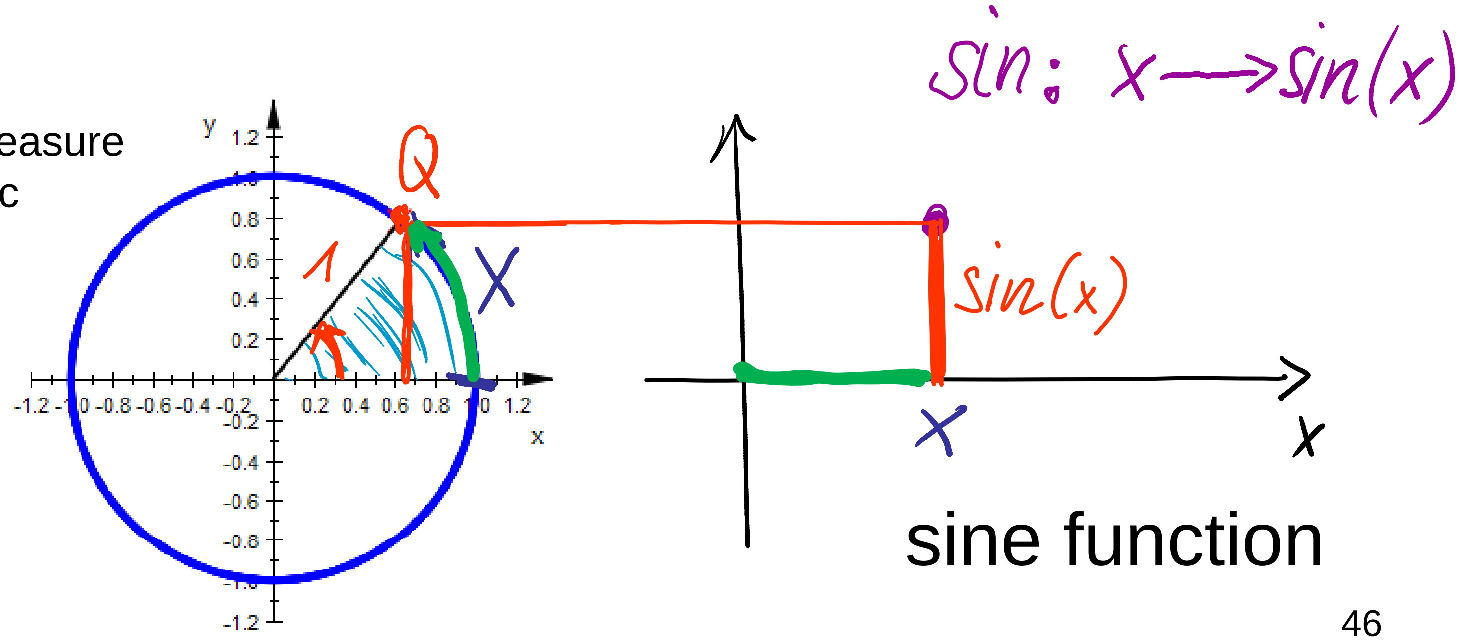
Sine Function

The angle x correlates with the ordinate of Q.

The function, which is able to do do, is the sine function.

unit circle

x = angle in radian measure
 = distance of the arc
    in the unit circle



Die Kosinus-Funktion

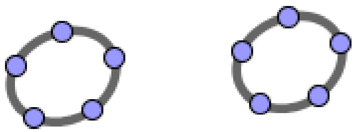
Dem Winkel x wird nun die ~~Ordinate~~ Abszisse von Q zugeordnet.

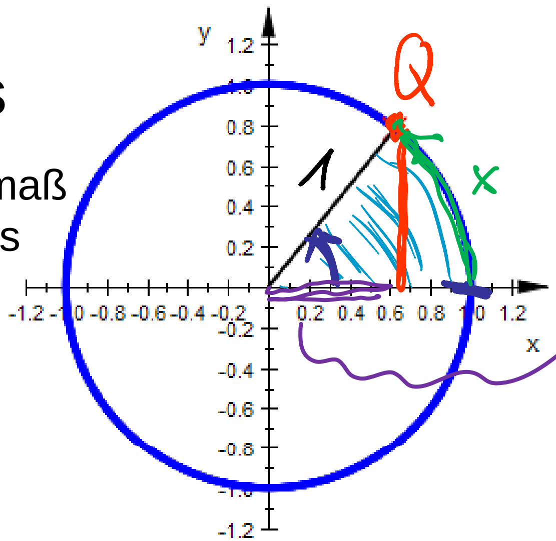
Die Funktion, die das leistet, heißt ~~Sinus~~ Kosinus-Funktion.

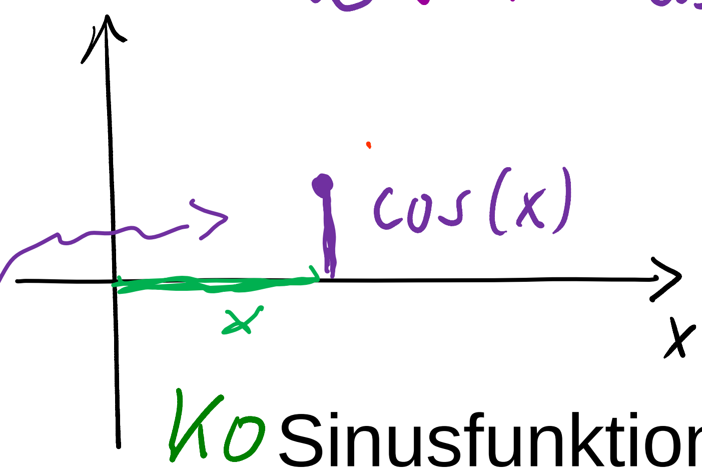
$\cos : x \longrightarrow \cos(x)$

Einheitskreis

x = Winkel im Bogenmaß
= Länge des Bogens im Einheitskreis







~~Sinusfunktion~~ Ko Sinusfunktion

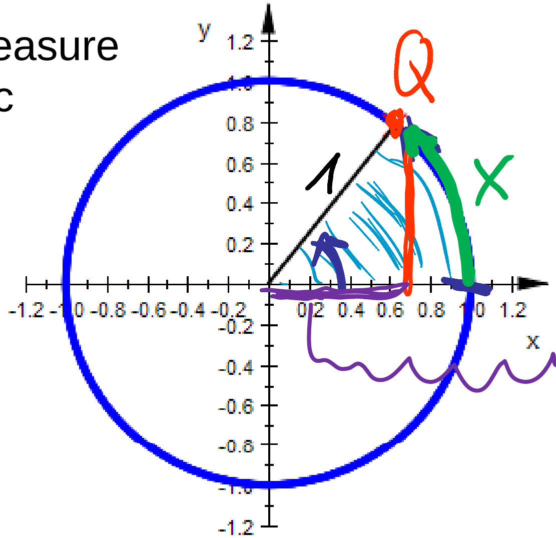
Cosine Function

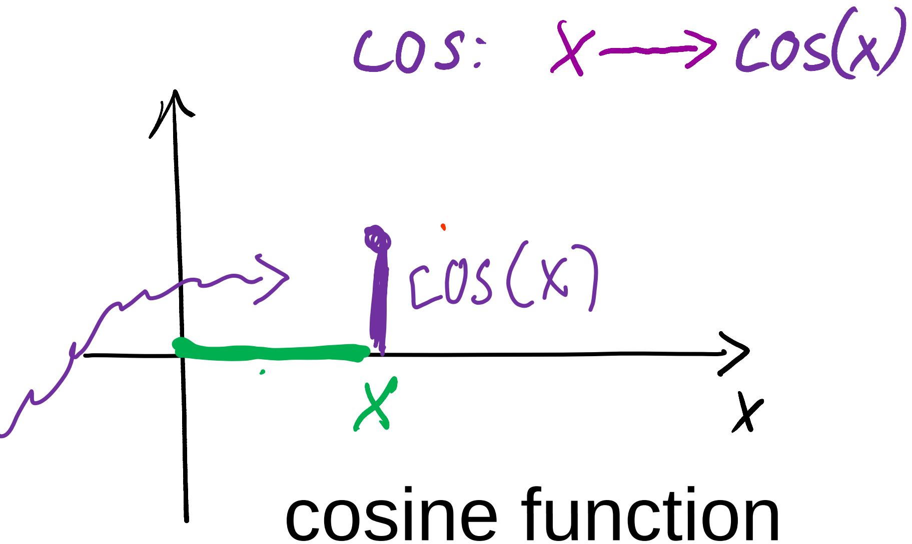
The angle x correlates with the ~~ordinate~~ abscissa of Q.

The function, which is able to do do, is the ~~sine~~ **cosine** function.

unit circle

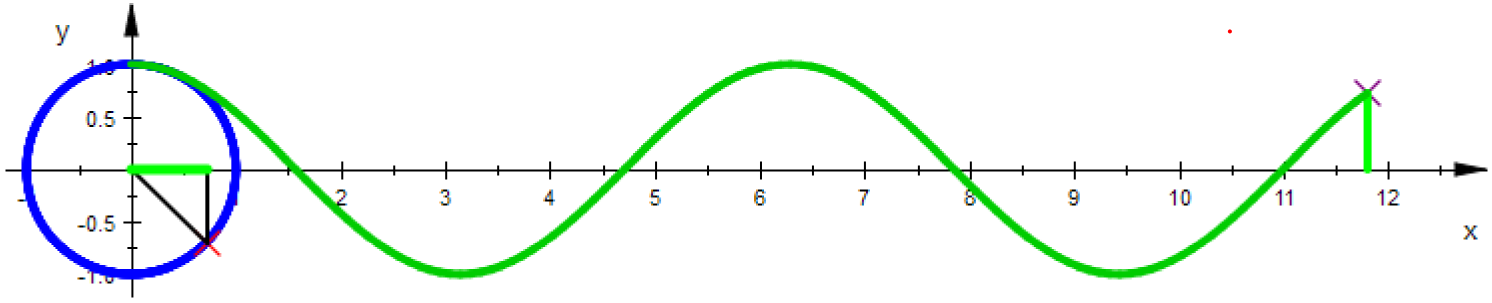
x = angle in radian measure
= distance of the arc in the unit circle



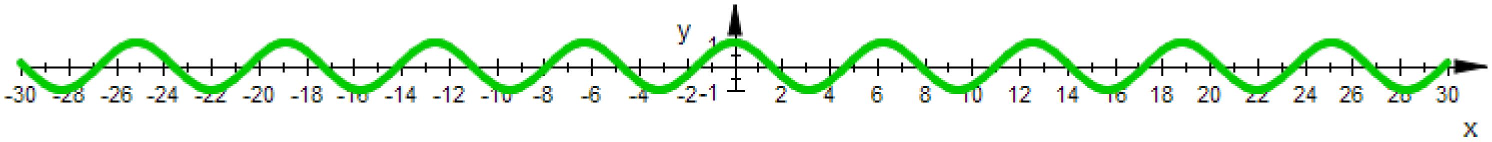


cosine function

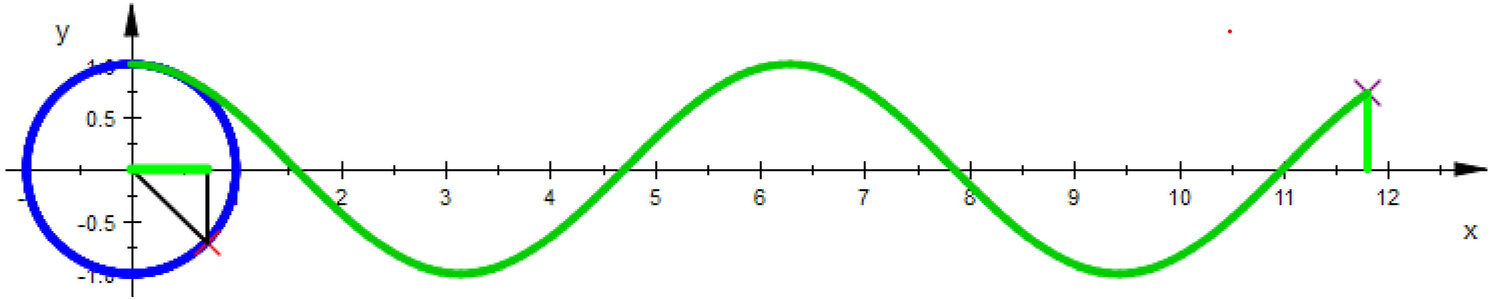
Eigenschaften der Kosinus-Funktion



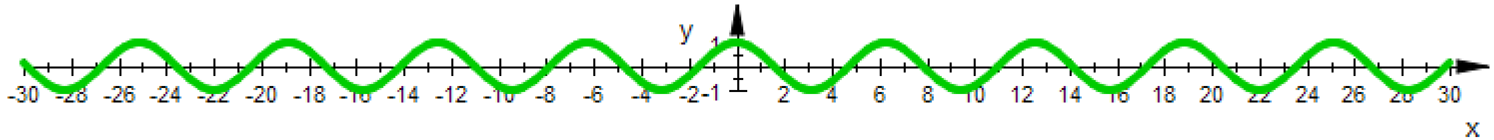
- Die Kosinus-Funktion ist periodisch.
- Die Periode ist 2π .
- Die Kosinuswerte liegen zwischen -1 und + 1.
- Die Kosinusbögen sind symmetrisch.
- Die Kosinuskurve ist symmetrisch zur y-Achse



Properties of the Cosine Function



- The cosine function is periodic.
- The period is 2π .
- The ordinates of the cosine are between -1 and + 1.
- The arcs of the cosine sind symmetrisch.
- The cosine curve is symmetric to the y-axis



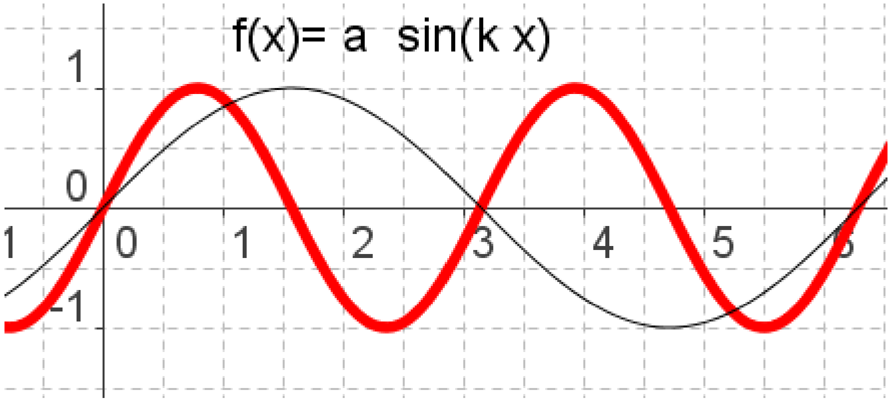
Sinus strecken und stauchen

Prof. Dr. Dörte Haftendorn, Leuphana Universität Lüneburg, 2013 http://www.leuphana.de/matheomnibus

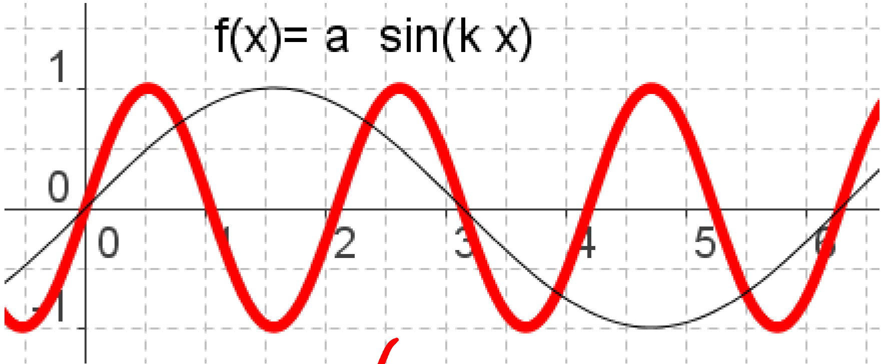
To Stretch and Compress the Sine Function

Funktionen strecken und stauchen

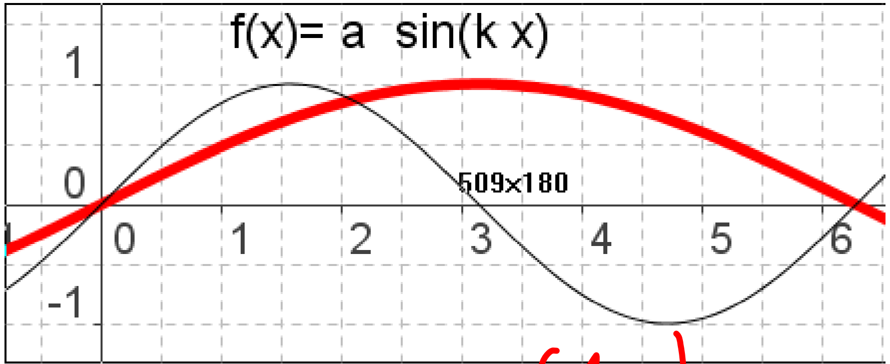
$y = \sin(2x)$



Ein Faktor direkt beim x sorgt für waagerechtes Strecken und Stauchen.



$y = \sin(3x)$



$y = \sin(\frac{1}{2}x)$

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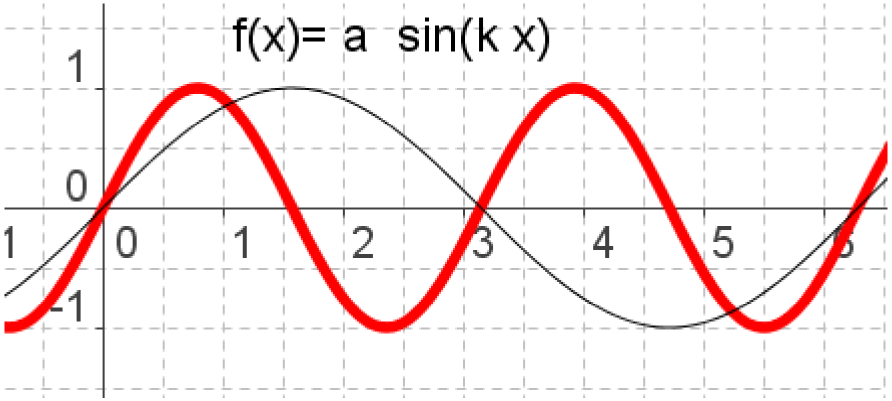
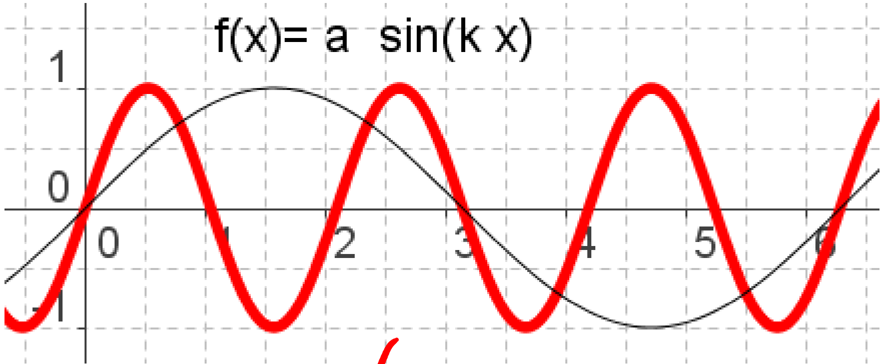
To Stretch and Compress the Functions

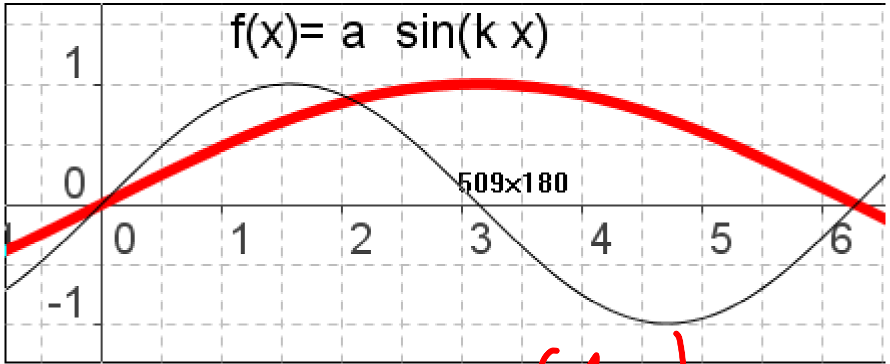
$y = \sin(2x)$



to stretch and compress
in the direction of the x-axis
you must put a factor to
x directly



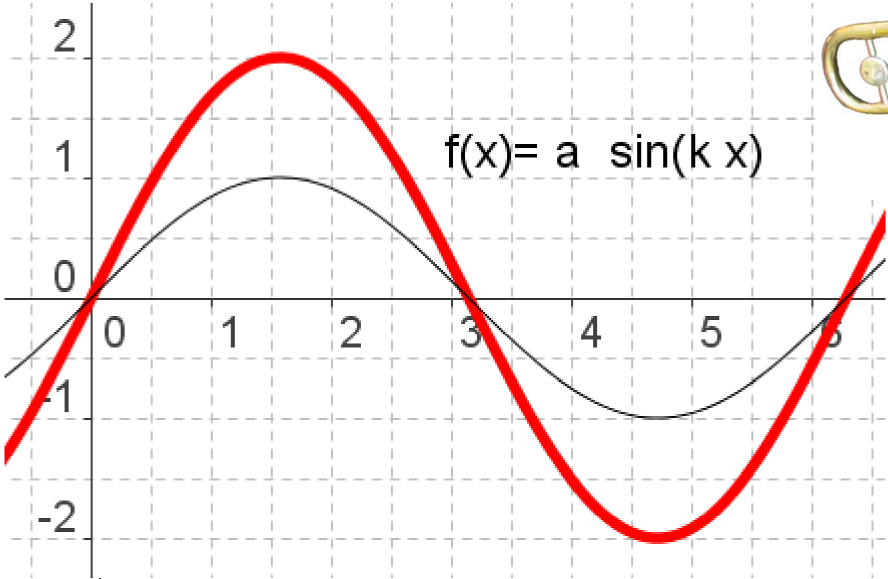
$y = \sin(3x)$

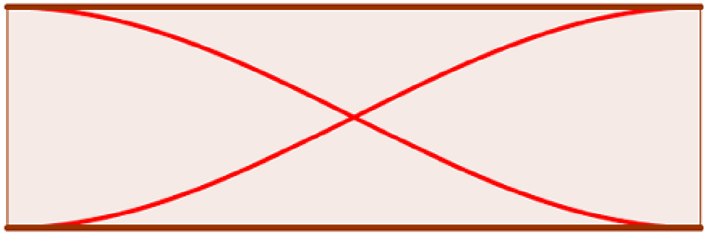


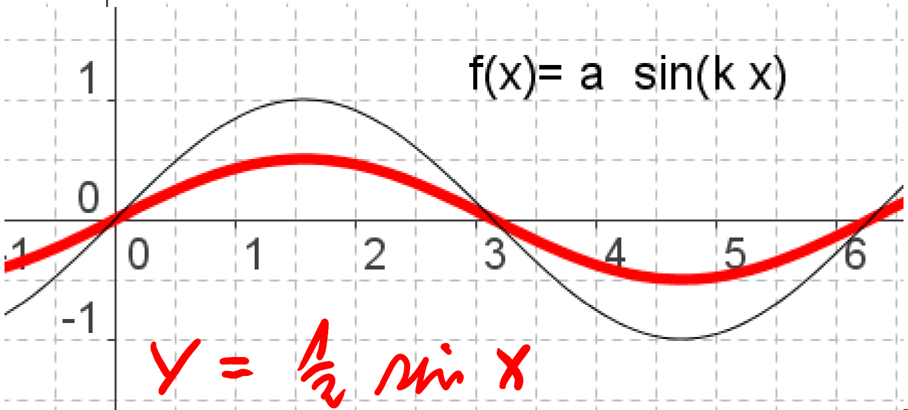
$y = \sin(\frac{1}{2}x)$

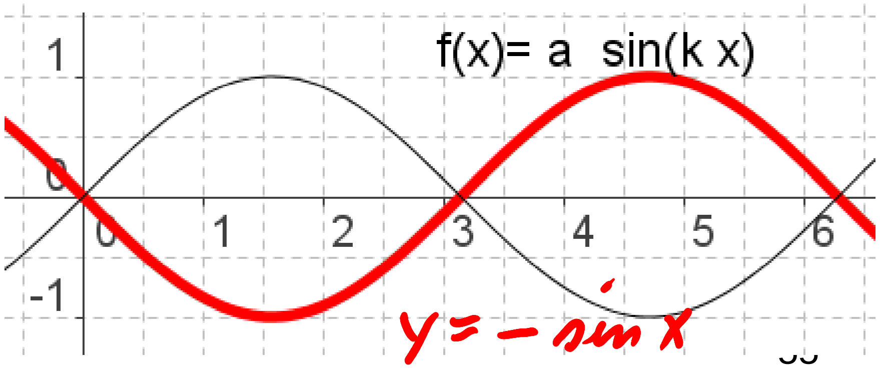
Funktionen strecken und stauchen

$y = 2 \sin x$



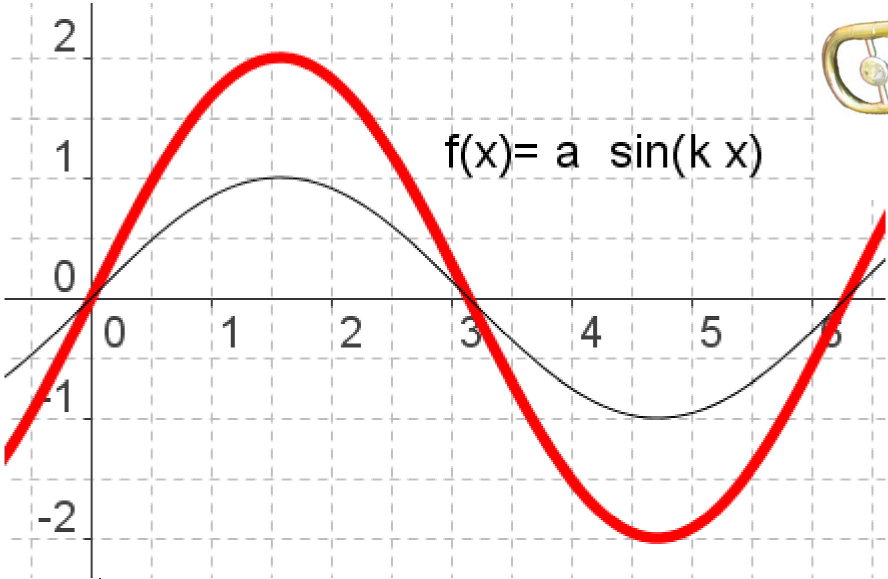


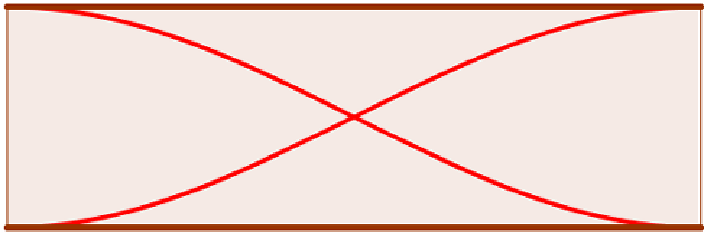


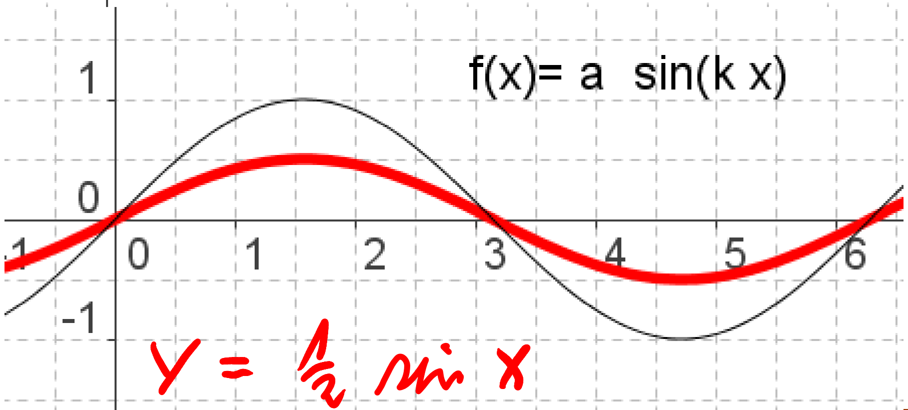


To Stretch and Compress the Functions

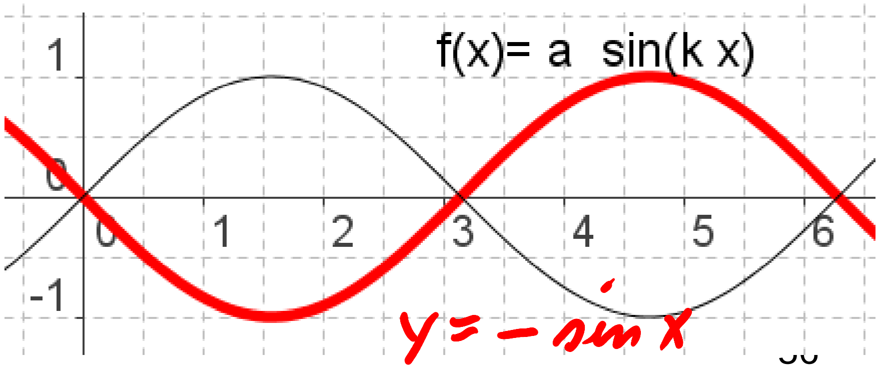
$y = 2 \sin x$







$y = \frac{1}{2} \sin x$

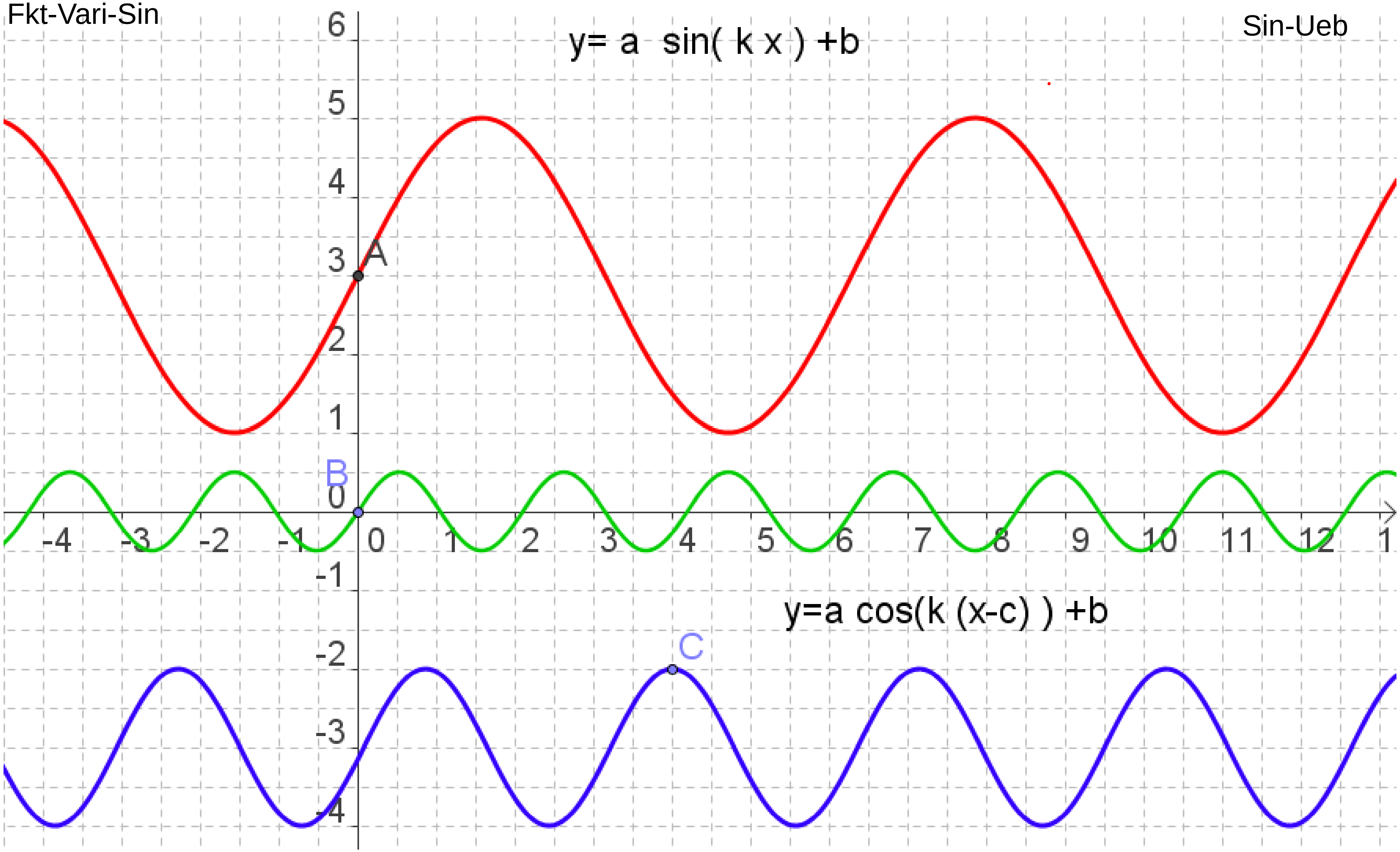


$y = -\sin x$

Funktionen variieren

Fkt-Vari-Sin

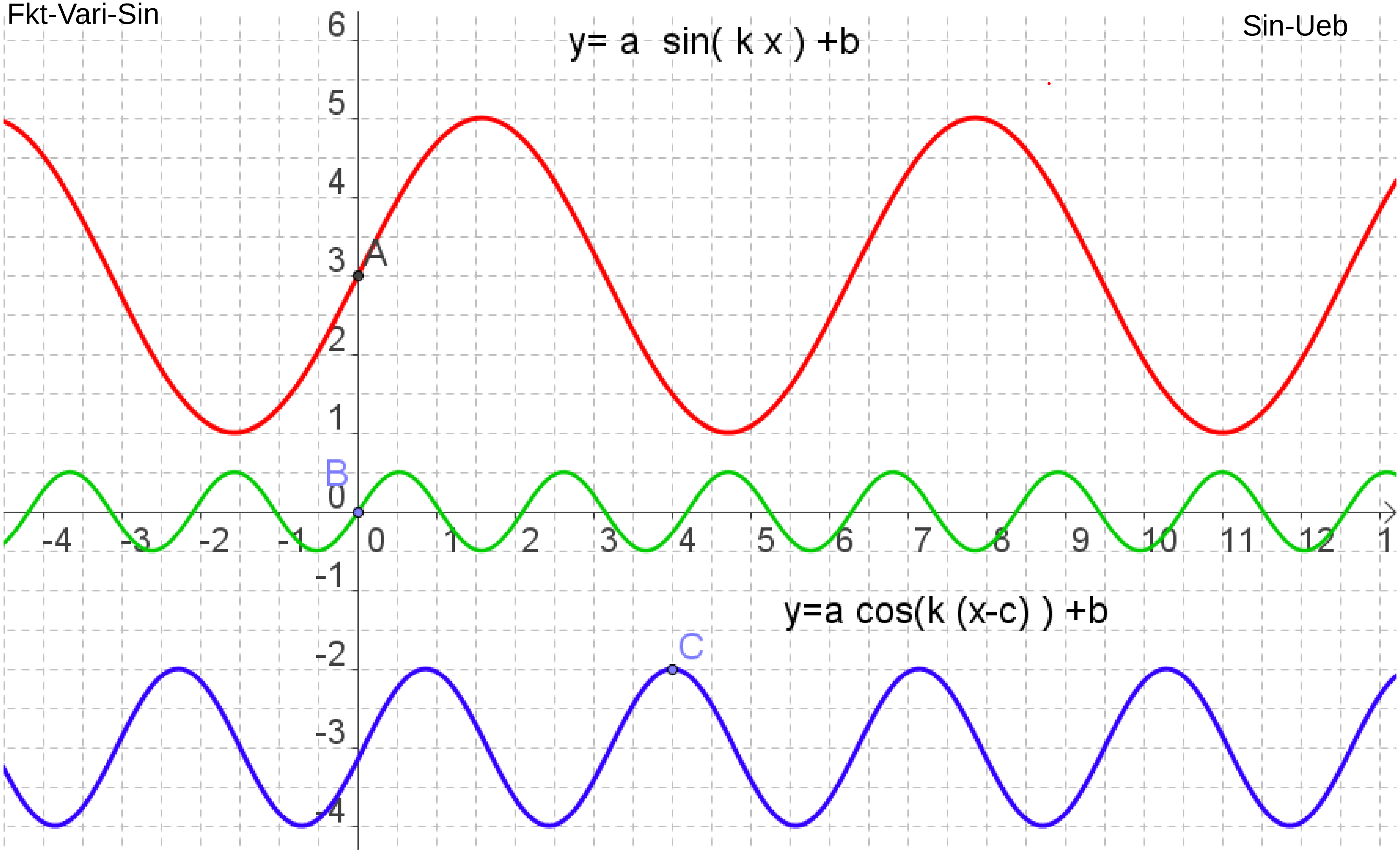
Sin-Ueb



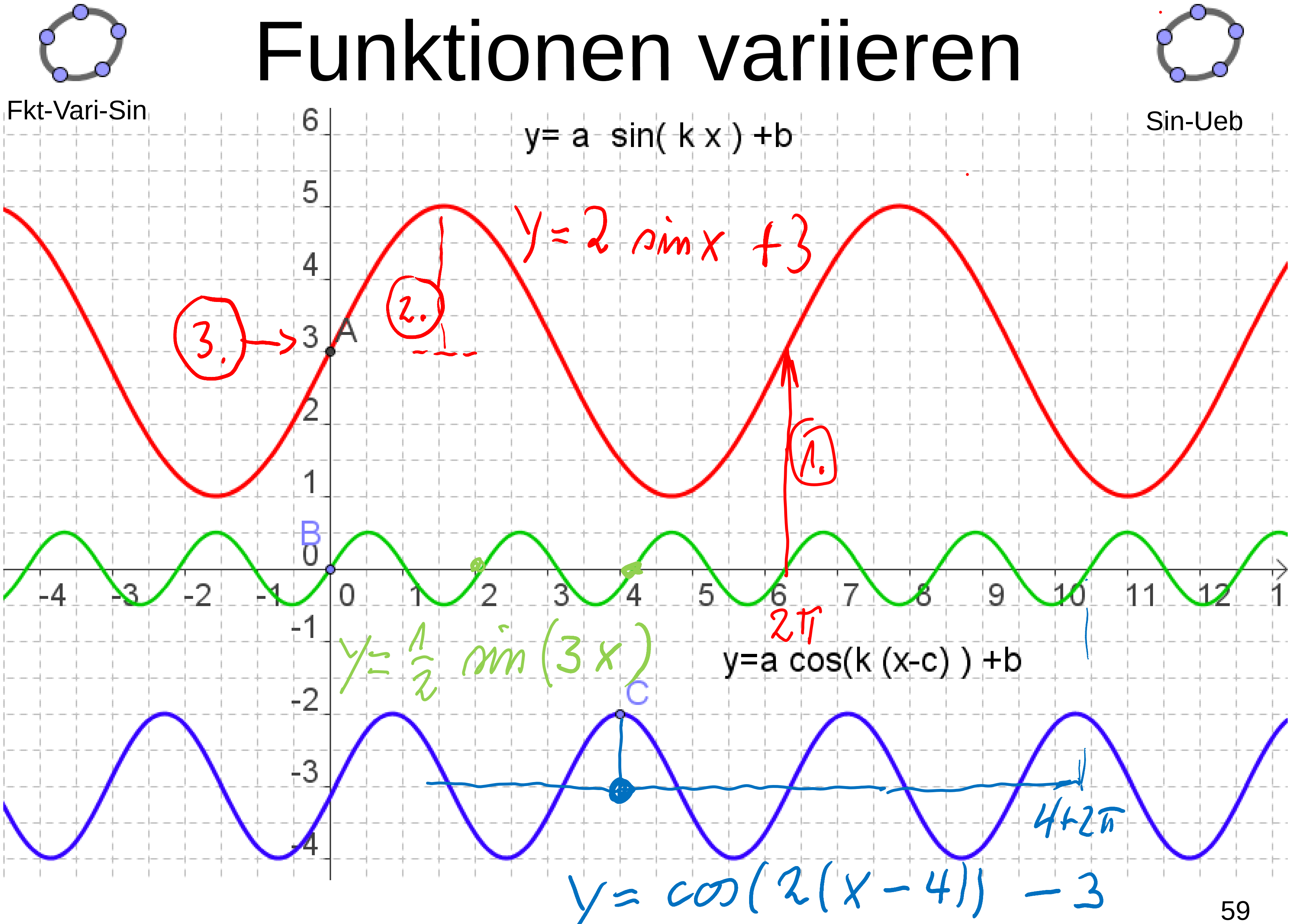
Variation of Functions

Fkt-Vari-Sin

Sin-Ueb



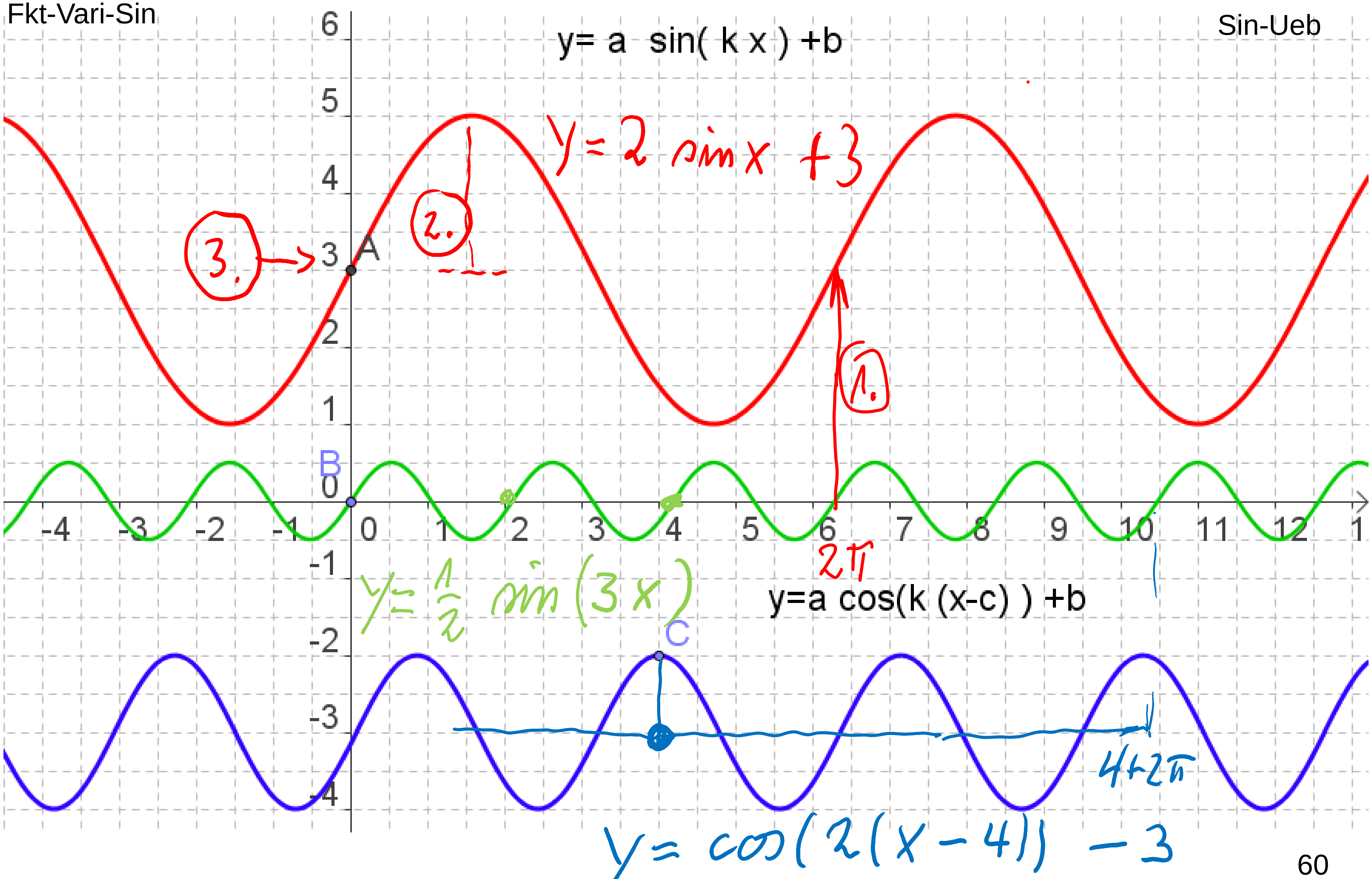
Funktionen variieren



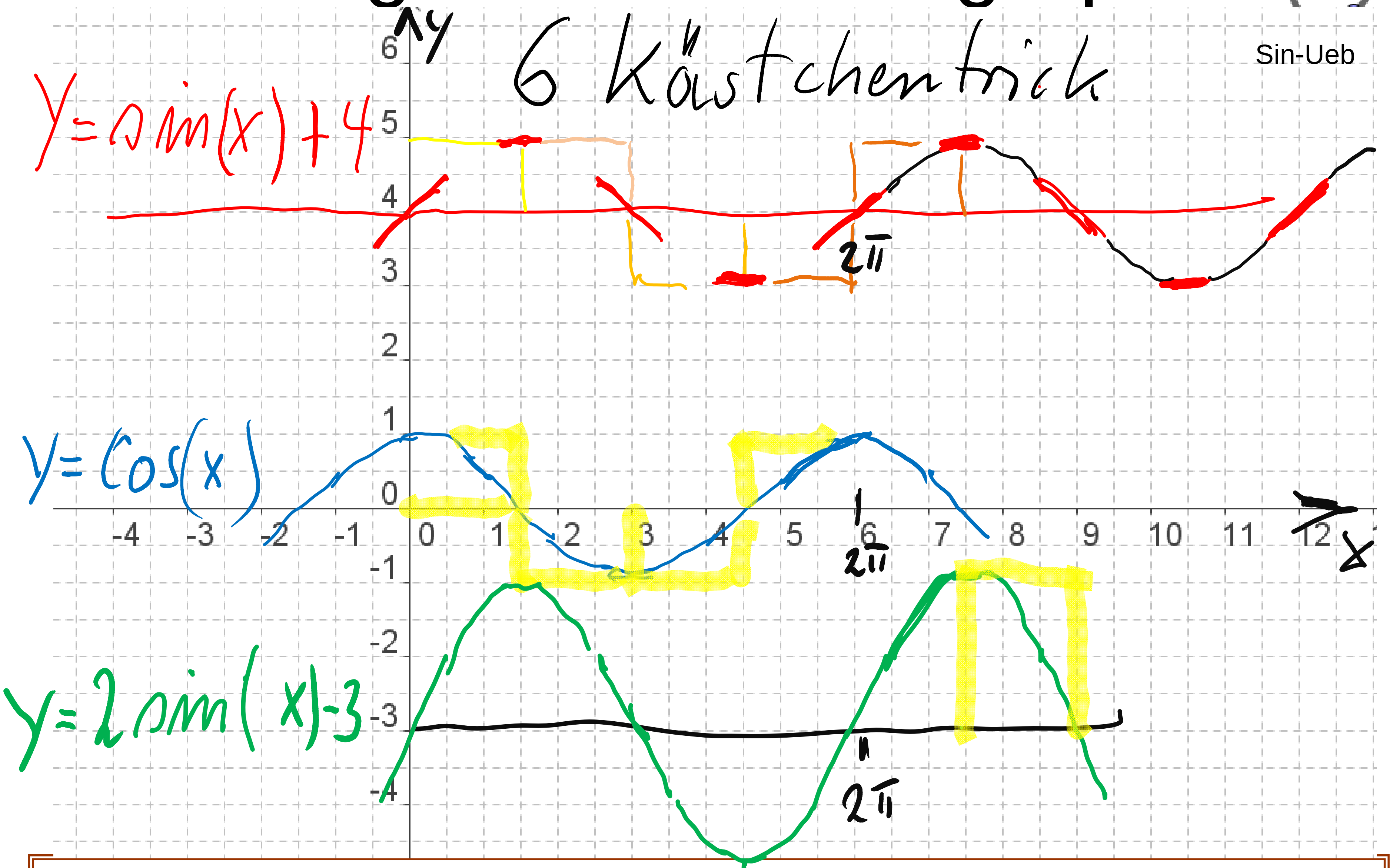
Variation of Functions

Fkt-Vari-Sin

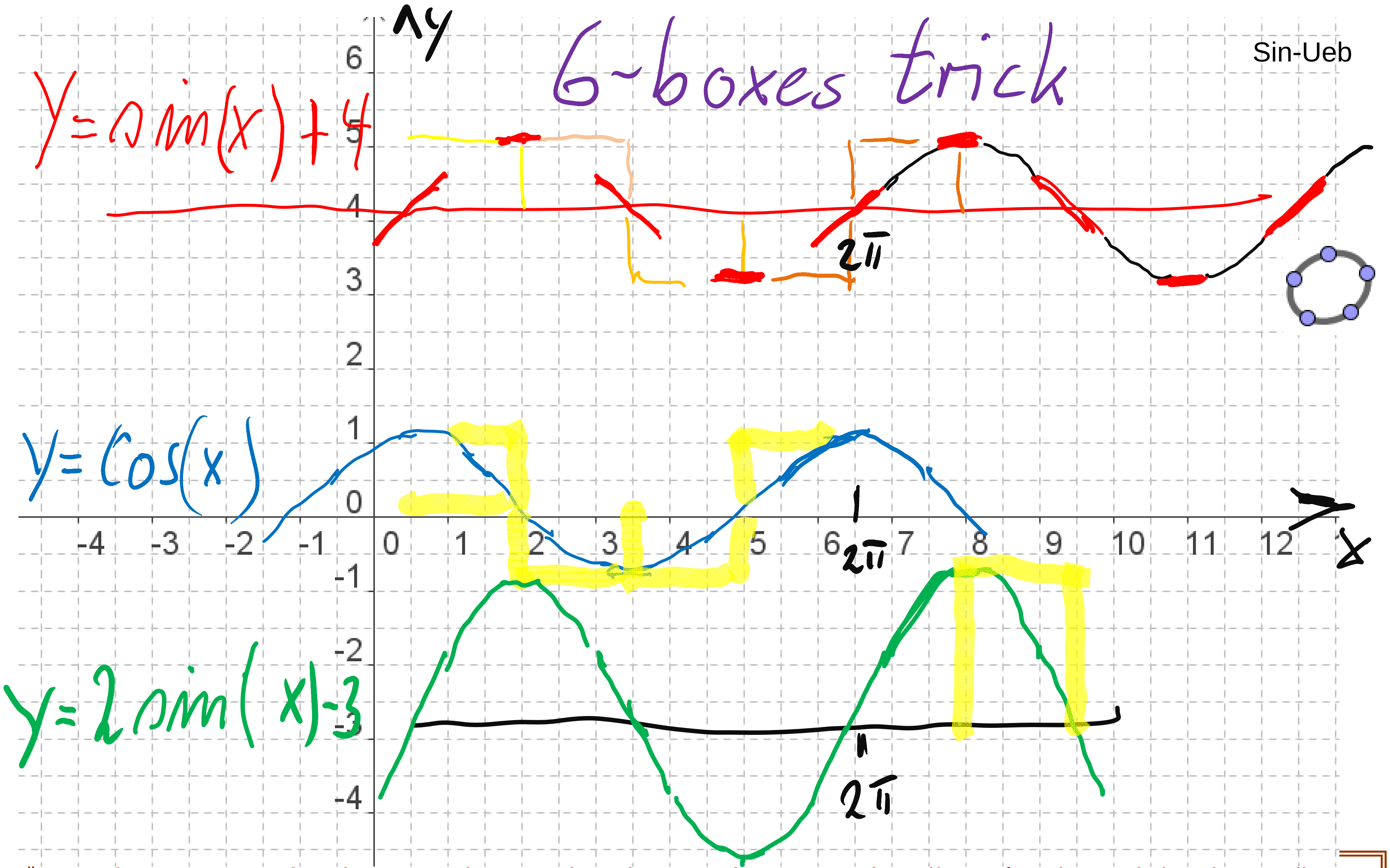
Sin-Ueb



Übung mit Funktionsgraphen



How to Sketch Sine and Cosine Function



Welle, wave