Polynome im Affenkasten

Polynome 3. Grades   Parabeln   Panther

Dritten Grades

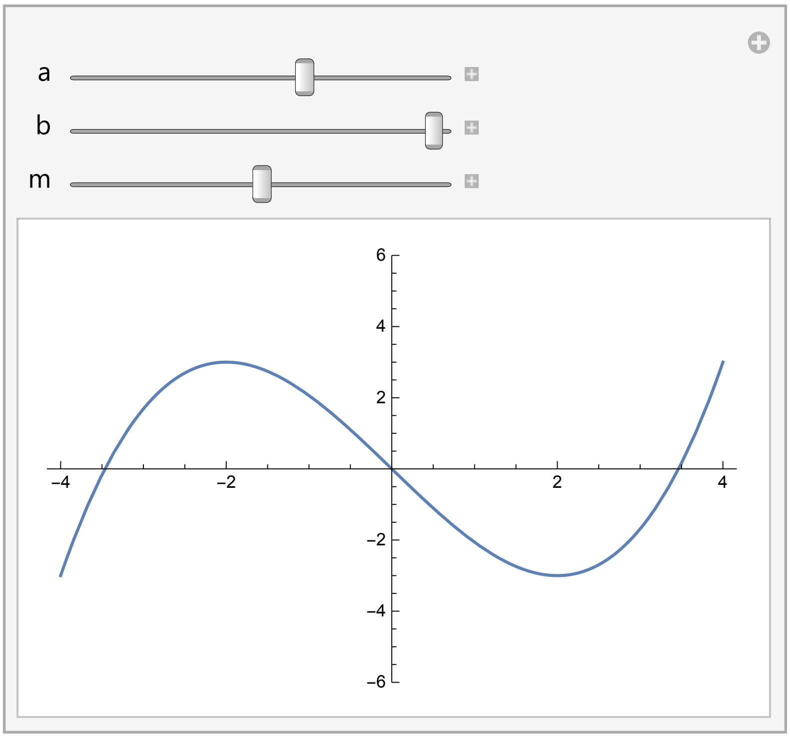
In[∘]:=
```
g[x_] := b/(2 a^3) x^3 + (m - 3 b/(2 a)) x
```

In[∘]:=
```
g[x]
```

Out[∘]= $\left(-\frac{3\,b}{2\,a} + m\right) x + \frac{b\,x^3}{2\,a^3}$

In[∘]:=
```
Manipulate[Plot[(-3 b/(2 a) + m) x + b x^3/(2 a^3), {x, -4, 4}, PlotRange → {-6, 6}],
 {{a, 2}, -8, 8}, {{b, 3}, -3, 3}, {{m, 0}, -2, 2}]
```

manipuliere   stelle Funktion graphisch dar   Koordinatenbereich der Graph

Out[∘]= 

In[∘]:=
```
h[x_] := r x^3 - s x; h[x]
```

Out[∘]= $-s\,x + r\,x^3$

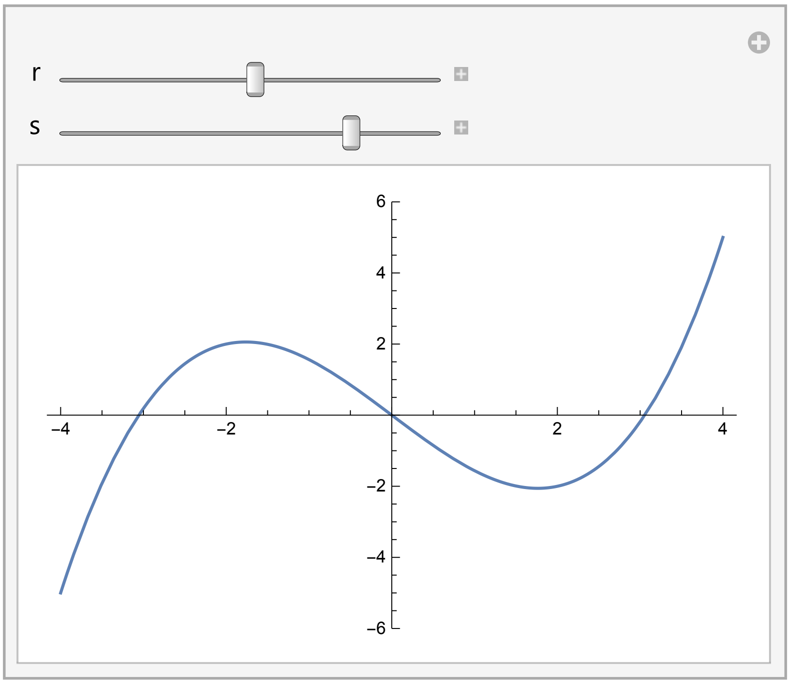
Fall $m = \frac{1}{2}$

In[∘]:=
```
{b/(2 a^3), m + (-3 b)/(2 a)} /. {a → 2, b → 3, m → 1/2}
```

Out[∘]= $\left\{\frac{3}{16}, -\frac{7}{4}\right\}$

In[∘]:= Manipulate[Plot[-s x + r x^3 , {x, -4, 4} , PlotRange → {-6, 6}],
manipuliere | stelle Funktion graphisch dar | Koordinatenbereich der Graph
{{r, 3/16}, -8, 8}, {{s, 7/4}, -3, 3}]

Out[∘]=



In[∘]:= Quit
beende Kernel

Fall1 m=0

In[∘]:= g[x]

In[∘]:= {a, b, r, s}

Out[∘]= {a, b, r, s}

In[∘]:= g[2 a] =.

In[∘]:= ? g

Global`g

$g[x\_] := r x^3 - s x$

In[∘]:= {h[a], h'[a], h[2 a]}

Out[∘]= $\{a^3 r - a s, 3 a^2 r - s, 8 a^3 r - 2 a s\}$

In[∘]:= h[a] == -b; h'[a] == 0; h[2 a] == b;

In[∘]:= Solve[{h[a] == -b, h'[a] == 0, h[2 a] == b}, {r, s}]
löse

Out[∘]= $\left\{\left\{r \to \frac{b}{2 a^3}, s \to \frac{3 b}{2 a}\right\}\right\}$

Fall 2 m<>0

In[∘]:=
```
eins = h[a] == m a - b
```
Out[∘]= $a^3 r - a s == -b + a m$

In[∘]:=
```
h'[x]
```
Out[∘]= $-s + 3 r x^2$

In[∘]:=
```
zwei = h'[a] == m
```
Out[∘]= $3 a^2 r - s == m$

In[∘]:=
```
h[2 a]
```
Out[∘]= $8 a^3 r - 2 a s$

In[∘]:=
```
drei = h[2 a] == 2 m a + b
```
Out[∘]= $8 a^3 r - 2 a s == b + 2 a m$

In[∘]:=
```
Solve[{eins, zwei, drei}, {a, b, m}]
```
löse

Out[∘]= $\{\{b \to 2 a^3 r, m \to 3 a^2 r - s\}\}$

In[∘]:=
```
Solve[{eins, zwei, drei}, {a, b}]
```
löse

Out[∘]=
$$\left\{\left\{a \to -\frac{\sqrt{m+s}}{\sqrt{3}\sqrt{r}}, b \to \frac{2}{9}\left(-\frac{\sqrt{3}\, m\sqrt{m+s}}{\sqrt{r}} - \frac{\sqrt{3}\, s\sqrt{m+s}}{\sqrt{r}}\right)\right\}, \left\{a \to \frac{\sqrt{m+s}}{\sqrt{3}\sqrt{r}}, b \to \frac{2}{9}\left(\frac{\sqrt{3}\, m\sqrt{m+s}}{\sqrt{r}} + \frac{\sqrt{3}\, s\sqrt{m+s}}{\sqrt{r}}\right)\right\}\right\}$$

In[∘]:=
```
Solve[{eins, zwei, drei}, {a, m}]
```
löse

Out[∘]=
$$\left\{\left\{a \to -\frac{\left(-\frac{1}{2}\right)^{1/3} b^{1/3}}{r^{1/3}}, m \to \frac{3 (-1)^{2/3} b^{2/3} r^{1/3}}{2^{2/3}} - s\right\}, \left\{a \to \frac{b^{1/3}}{2^{1/3} r^{1/3}}, m \to \frac{3 b^{2/3} r^{1/3}}{2^{2/3}} - s\right\}, \left\{a \to \frac{(-1)^{2/3} b^{1/3}}{2^{1/3} r^{1/3}}, m \to \frac{1}{2}\left(-3 (-2)^{1/3} b^{2/3} r^{1/3} - 2 s\right)\right\}\right\}$$

Tests für das Buch

In[∘]:=
```
Quit
```
beende Kernel

In[∘]:=
```
p[x_] := t (x + 3 a) x^2
```

In[∘]:=
```
p[x]
```
Out[∘]= $t x^2 (3 a + x)$

In[∘]:= $b = \frac{p[a]}{2}$

Out[∘]= $\frac{p[a]}{2}$

In[∘]:= `b =.`

In[∘]:= $t = \frac{b}{2 a^3}$

Out[∘]= $\frac{b}{2 a^3}$

In[∘]:= `f[x_] := p[x - a] - b ; f[x]`

Out[∘]= $-b + \frac{b (-a + x)^2 (2 a + x)}{2 a^3}$

In[∘]:= `Collect[Expand[f[x]], x]`
gruppier··· multipliziere aus

Out[∘]= $-\frac{3 b x}{2 a} + \frac{b x^3}{2 a^3}$

Flächen

In[∘]:= `p[x]`

Out[∘]= $\frac{b x^2 (3 a + x)}{2 a^3}$

In[∘]:= `{a, b} = {2, 3}; Integrate[`$\frac{b x^2 (3 a + x)}{2 a^3}$`, {x, -3 a, a}]`
integriere

`{a, b} =.;`

Out[∘]= 24

In[∘]:= `Integrate[`$\frac{b x^2 (3 a + x)}{2 a^3}$`, {x, -3 a, a}]`
integriere

Out[∘]= 4 a b

In[∘]:= `Integrate[`$\frac{b x^2 (3 a + x)}{2 a^3}$`, x]`
integriere

Out[∘]= $\frac{b \left(a x^3 + \frac{x^4}{4}\right)}{2 a^3}$

In[∘]:= `Integrate[`$\frac{b x^2 (3 a + x)}{2 a^3}$`, {x, -3 a, 0}]`
integriere

Out[∘]= $\frac{27 a b}{8}$

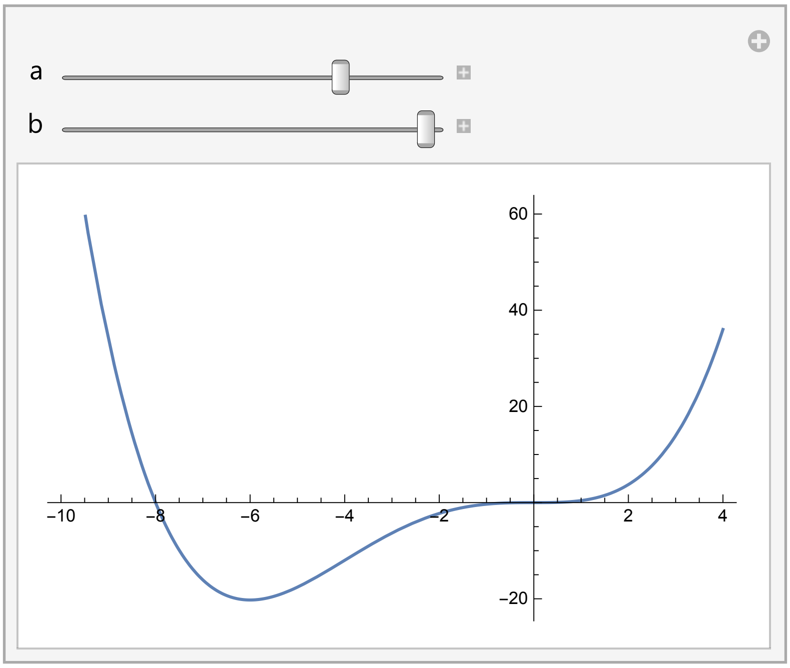
In[∘]:= $\frac{27\,a\,b}{8 \times 4\,a\,b}$

Out[∘]= $\frac{27}{32}$

In[∘]:= $\frac{27\,a\,b}{8 \times 6\,a\,b}$

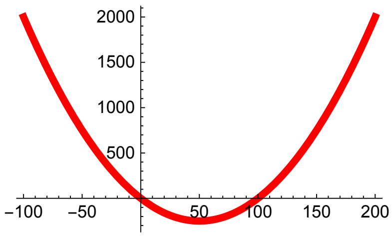
Out[∘]= $\frac{9}{16}$

In[∘]:=
```
Manipulate[Plot[b (a x^3 + x^4/4) / (2 a^3), {x, -10, 4}], {{a, 2}, -4, 4}, {{b, 3}, -3, 3}]
```
manipuliere · stelle Funktion graphisch dar

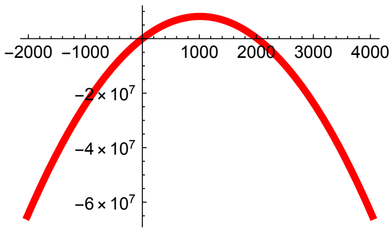
Out[∘]=


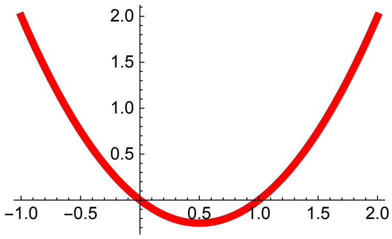
Parabeln

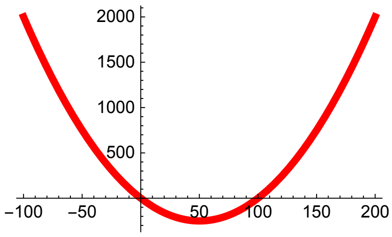
In[∘]:=
```
a = 0.1; b = 100;
Plot[a x (x - b), {x, -b, 2 b}, PlotRange → All,
 PlotStyle → {Thickness[0.02], Color = Red}]
{a, b} =.
```
stelle Funktion graphisch dar · Koordinatenb··· · alle · Dicke · rot

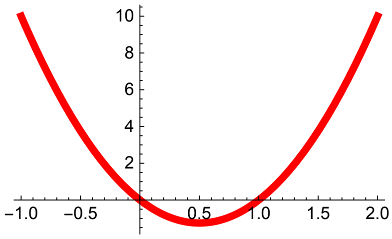
Out[∘]=


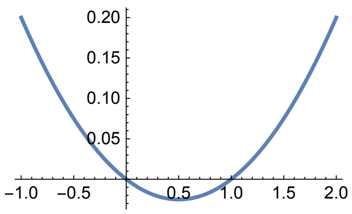
Out[∘]= {Null, Null}

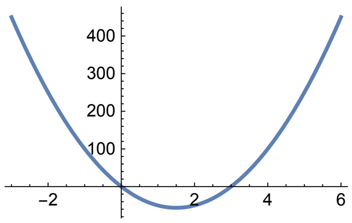

−2000 −1000 1000 2000 3000 4000
−2×10^7
−4×10^7
−6×10^7

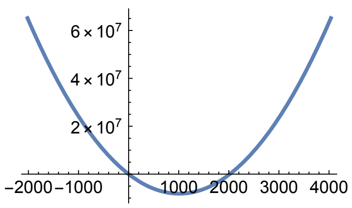

2.0
1.5
1.0
0.5
−1.0 −0.5 0.5 1.0 1.5 2.0


2000
1500
1000
500
−100 −50 50 100 150 200


10
8
6
4
2
−1.0 −0.5 0.5 1.0 1.5 2.0


0.20
0.15
0.10
0.05
−1.0 −0.5 0.5 1.0 1.5 2.0


400
300
200
100
−2 2 4 6


6×10^7
4×10^7
2×10^7
−2000 −1000 1000 2000 3000 4000

Pantherkäfig

In[∘]:=
```
Quit
```
beende Kernel

In[∘]:=
```
Expand[(x + r) x (x - w) (x - w - r)]
```
multipliziere aus

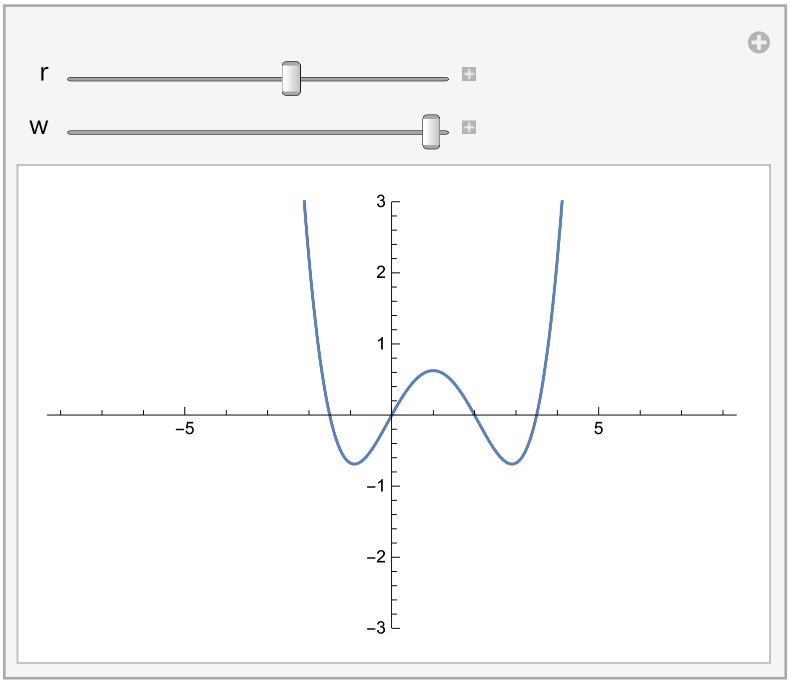
Out[∘]= $r^2 w x + r w^2 x - r^2 x^2 - r w x^2 + w^2 x^2 - 2 w x^3 + x^4$

In[∘]:=
```
Collect[%, x]
```
gruppiere Koeffizienten

Out[∘]= $(r^2 w + r w^2) x + (-r^2 - r w + w^2) x^2 - 2 w x^3 + x^4$

In[∘]:=
```
Manipulate[Plot[0.1 ((r^2 w + r w^2) x + (-r^2 - r w + w^2) x^2 - 2 w x^3 + x^4),
  {x, -8, 8}, PlotRange → {-3, 3}], {{r, 1.5}, -8, 8}, {{w, 2}, -2, 2}]
```
manipuliere | stelle Funktion graphisch dar | Koordinatenbereich der Graphik

Out[∘]=



In[∘]:= $g[x\_] := t\left(\frac{1}{12} x^4 - \frac{w}{6} x^3 + d\,x\right);\; g[x]$

Out[∘]= $t\left(d\,x - \frac{w\,x^3}{6} + \frac{x^4}{12}\right)$

Bedingung für Symmetrie

In[∘]:=
```
g[w]
```

Out[∘]= $t\left(d\,w - \frac{w^4}{12}\right)$

In[∘]:= $gs[x\_] := t\left(\frac{1}{12}x^4 - \frac{w}{6}x^3 + \frac{w^3}{12}\ x\right);\ gs[x]$

Out[∘]= $t\left(\frac{w^3\,x}{12} - \frac{w\,x^3}{6} + \frac{x^4}{12}\right)$

In[∘]:= `Factor[%]`
faktorisiere

$\frac{1}{12}\,t\,(w - x)\,x\,(w^2 + w\,x - x^2)\ s$

Polynome 4. Grades aufbauen

In[∘]:= `p[x_] := t x (x - w); p[x]`

Out[∘]= $t\,x\,(-w + x)$

In[∘]:= `f[x_] := Integrate[p[x], x]; f[x]`
integriere

Out[∘]= $t\left(-\frac{w\,x^2}{2} + \frac{x^3}{3}\right)$

In[∘]:= $\frac{t}{6}\,x\text{^}2\,(2\,x - 3\,w) + t\,d$

$d\,t + \frac{1}{6}\,t\,x^2\,(-3\,w + 2\,x)$

Dieses ist die allgemeine Form.

Verschieben Wendepunkt in den Ursprung

In[∘]:= $t\,d - \frac{1}{2}\,t\,w\,x^2 + \frac{t\,x^3}{3}$ /. $x \to x + \frac{w}{2}$ // Expand
multipliziere aus

Out[∘]= $d\,t - \frac{t\,w^3}{12} - \frac{1}{4}\,t\,w^2\,x + \frac{t\,x^3}{3}$

Wenn dieses punktsymmetrisch zum Ursprung sein soll, muss d passende gewählt werden

In[∘]:= $d\,t - \frac{t\,w^3}{12} - \frac{1}{4}\,t\,w^2\,x + \frac{t\,x^3}{3}$ /. $d \to \frac{w^3}{12}$

Out[∘]= $-\frac{1}{4}\,t\,w^2\,x + \frac{t\,x^3}{3}$

Also: Punktsymmetrisch zu $\left(\frac{w}{2},0\right)$

In[∘]:= $d\,t + \frac{1}{6}\,t\,x^2\,(-3\,w + 2\,x)$ /. $d \to \frac{w^3}{12}$ // Expand
multipliziere aus

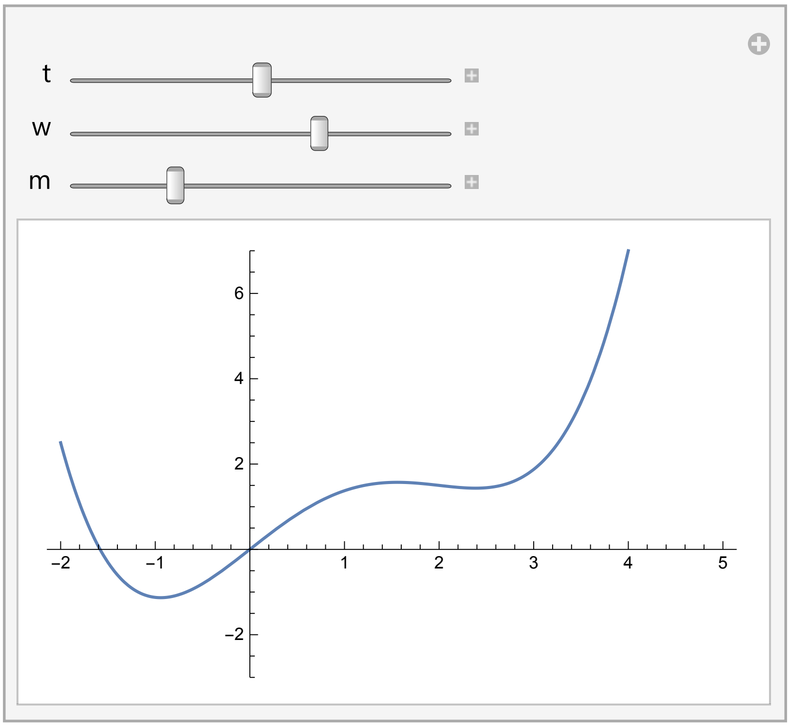
Out[∘]= $\frac{t\,w^3}{12} - \frac{1}{2}\,t\,w\,x^2 + \frac{t\,x^3}{3}$

Dazu gehört das Polynom 4. Grades

In[ ]:= $\text{Integrate}\left[\frac{t\,w^3}{12} - \frac{1}{2}\,t\,w\,x^2 + \frac{t\,x^3}{3},\ x\right]$
integriere

Out[ ]= $\frac{1}{12}\,t\,w^3\,x - \frac{1}{6}\,t\,w\,x^3 + \frac{t\,x^4}{12}$

In[ ]:= $\text{Manipulate}\left[\text{Plot}\left[\frac{1}{12}\,t\,w^3\,x - \frac{1}{6}\,t\,w\,x^3 + \frac{t\,x^4}{12} + m\,x,\ \{x, -2, 5\},\ \text{PlotRange} \to \{-3, 7\}\right],\ \left\{\left\{t, \frac{3}{2}\right\}, 0, 3\right\},\ \{\{w, 2\}, 0, 3\},\ \{\{m, 0.5\}, 0, 3\}\right]$
manipuliere | stelle Funktion graphisch dar | Koordinatenbereich der Graph

Out[ ]=


In[ ]:= $\frac{t}{12}\left(x^4 - 2\,w\,x^3 + w^3\,x\right)$

Out[ ]= $\frac{1}{12}\,t\left(w^3\,x - 2\,w\,x^3 + x^4\right)$

Veränderung dann als Scherung durch Addition von y= m x