Lösung der DGL $y' = y^2 - x$

Buch:Höhere Mathematik sehen und verstehen, Haftendorn, Riebesehl, Dammer, Springer Spektrum, Feb. 2021

Datei Lösung-y’=y2-x.nb zu Abschnitt 5.6.5 Seite 412 , Abb. 5.45

```
sol = DSolve[y'[x] == y[x]^2 - x, y[x], x] // Simplify
```

$$\left\{\left\{y[x] \to -\left(\text{BesselJ}\left[-\frac{1}{3}, \frac{2}{3} i x^{3/2}\right] C[1] + i x^{3/2} \left(2 \text{BesselJ}\left[-\frac{2}{3}, \frac{2}{3} i x^{3/2}\right] + \left(\text{BesselJ}\left[-\frac{4}{3}, \frac{2}{3} i x^{3/2}\right] - \text{BesselJ}\left[\frac{2}{3}, \frac{2}{3} i x^{3/2}\right]\right) C[1]\right)\right) \Big/ \left(2 x \left(\text{BesselJ}\left[\frac{1}{3}, \frac{2}{3} i x^{3/2}\right] + \text{BesselJ}\left[-\frac{1}{3}, \frac{2}{3} i x^{3/2}\right] C[1]\right)\right)\right\}\right\}$$

```
yt = y[x] /. sol[[1]] /. BesselJ[ν_, x_] → i^ν BesselI[ν, x/i] // FullSimplify
```

$$\frac{\sqrt{x}\left(-\text{BesselI}\left[-\frac{2}{3}, \frac{2 x^{3/2}}{3}\right] + (-1)^{2/3} \text{BesselI}\left[\frac{2}{3}, \frac{2 x^{3/2}}{3}\right] C[1]\right)}{\text{BesselI}\left[\frac{1}{3}, \frac{2 x^{3/2}}{3}\right] - (-1)^{2/3} \text{BesselI}\left[-\frac{1}{3}, \frac{2 x^{3/2}}{3}\right] C[1]}$$

```
(-1)^(2/3) // N
```

$-0.5 + 0.866025\, i$

```
yt = yt /. {(-1)^(2/3) → (-1 + √3 i)/2, C[1] -> (1 + √3 i)/2 c} // FullSimplify
```

$$-\frac{\sqrt{x}\left(\text{BesselI}\left[-\frac{2}{3}, \frac{2 x^{3/2}}{3}\right] + c\, \text{BesselI}\left[\frac{2}{3}, \frac{2 x^{3/2}}{3}\right]\right)}{c\, \text{BesselI}\left[-\frac{1}{3}, \frac{2 x^{3/2}}{3}\right] + \text{BesselI}\left[\frac{1}{3}, \frac{2 x^{3/2}}{3}\right]}$$

Das folgende Ergebnis erlaubt es, c aus der Anfangsbedingung in x=0 zu bestimmen:

$$\text{Limit}\left[-\frac{\sqrt{x}\left(\text{BesselI}\left[-\frac{2}{3}, \frac{2 x^{3/2}}{3}\right] + c\, \text{BesselI}\left[\frac{2}{3}, \frac{2 x^{3/2}}{3}\right]\right)}{c\, \text{BesselI}\left[-\frac{1}{3}, \frac{2 x^{3/2}}{3}\right] + \text{BesselI}\left[\frac{1}{3}, \frac{2 x^{3/2}}{3}\right]}, x \to 0\right]$$

$$-\frac{3^{1/3}\, \text{Gamma}\left[\frac{2}{3}\right]}{c\, \text{Gamma}\left[\frac{1}{3}\right]}$$

```
yt /. c → 0
```

$$-\frac{\sqrt{x}\, \text{BesselI}\left[-\frac{2}{3}, \frac{2 x^{3/2}}{3}\right]}{\text{BesselI}\left[\frac{1}{3}, \frac{2 x^{3/2}}{3}\right]}$$

```
yt2 = (yt /. c → 1/c // Simplify) /. c → 0
```

$$-\frac{\sqrt{x}\,\text{BesselI}\left[\frac{2}{3}, \frac{2x^{3/2}}{3}\right]}{\text{BesselI}\left[-\frac{1}{3}, \frac{2x^{3/2}}{3}\right]}$$

$$a = \frac{-1 - i\sqrt{3}}{2};$$

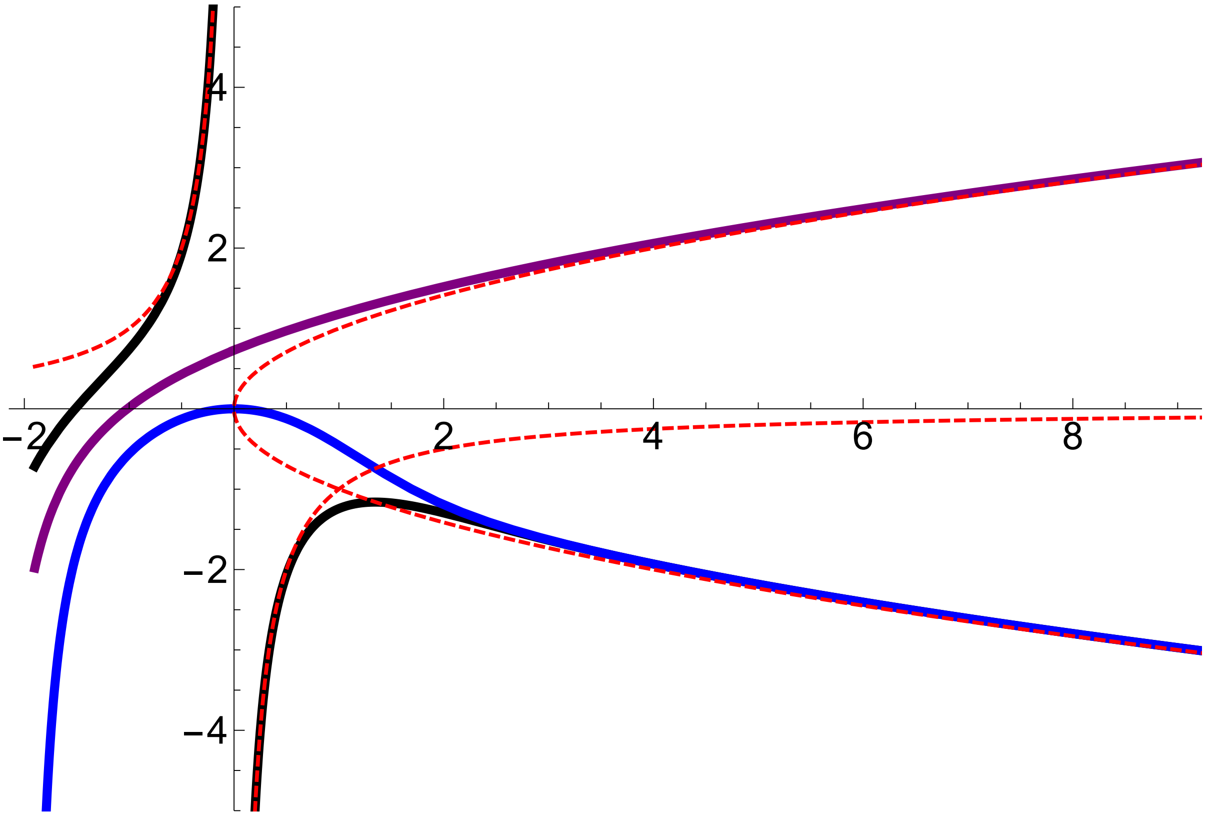
```
(yt /. c → -1 // FullSimplify // PowerExpand)
```

$$-\frac{\text{AiryAiPrime}[x]}{\text{AiryAi}[x]}$$

0.729011132947227`

0.7290111

```
Plot[{yt /. c → 0, -AiryAiPrime[x]/AiryAi[x], yt2, -√x, √x, -1/x},
 {x, -1.9, 10}, PlotRange → {-5, 5},
 PlotStyle → {{Black, Thickness[0.007]}, {Purple, Thickness[0.007]},
   {Blue, Thickness[0.007]}, {Dashed, Red, Thick}, {Dashed, Red, Thick},
   {Dashed, Red, Thick}}, BaseStyle → FontSize → 20, Exclusions → {x = 0}]
```



Das folgende, auch mit exakten Zahlen gezeichnete Bild, als Skurrilität.

```
Plot[{yt /. c → 0, (yt /. c → -1) // Re, yt2, - √x , √x , - 1/x},
 {x, -1.9, 10}, PlotRange → {-5, 5},
 PlotStyle → {{Black, Thickness[0.007]}, {Purple, Thickness[0.005]},
   {Blue, Thickness[0.007]}, {Dashed, Red, Thick}, {Dashed, Red, Thick},
   {Dashed, Red, Thick}}, BaseStyle → FontSize → 20, Exclusions → {x = 0}]
```