Höhere Mathematik sehen und verstehen,

Haftendorn, Riebesehl, Aug. 2021 Interaktive Version mit kostenlosem Mathematica Player

Beziersplines+B-Splines

Bézierkurven

In[∘]:=
```
b0[t_] := (1 - t)^3; b1[t_] := 3 t (1 - t)^2; b2[t_] := 3 t^2 (1 - t); b3[t_] := t^3
```

In[∘]:=
```
b0[t] // Expand
          multiplizi
b1[t] // Expand
          multiplizi
b2[t] // Expand
          multiplizi
b3[t] // Expand
          multiplizi
```

Out[∘]= $1 - 3 t + 3 t^2 - t^3$

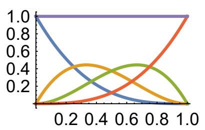
Out[∘]= $3 t - 6 t^2 + 3 t^3$

Out[∘]= $3 t^2 - 3 t^3$

Out[∘]= t^3

In[∘]:=
```
Plot[{b0[t], b1[t], b2[t], b3[t], b0[t] + b1[t] + b2[t] + b3[t]}, {t, 0, 1}]
stelle Funktion graphisch dar
```

Out[∘]=



“Didaktische” B-Splines mit Polynomen 4.Grades

Die echten B-Splines sehen aus wie Polynome 4. Grades, mit zwei doppelten Nullstellen.
Da später Summe 1 verwirklicht werden soll, ist hier ein Faktor a0 vorgesehen.

In[∘]:=
```
V[0, t_] := Piecewise[{{a0 t^2 (t - 4)^2, 0 ≤ t ≤ 4}, {0 a0, True}}];
            stückweise                                    wahr
```

Rekursive Definition

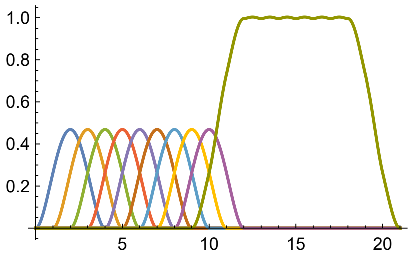
```
a0 =.; V[i_, t_] := V[i - 1, t - 1] (*Stückweise Polynome 4, Grades.*)
V[11, t]
```

Out[∘]= $\begin{cases} a0\,(-15+t)^2\,(-11+t)^2 & 0 \le -11+t \le 4 \\ 0 & \text{True} \end{cases}$

```
In[∘]:= su[t_] := V[0, t ] + V[1, t] + V[2, t ] + V[3, t ] +
  V[4, t] + V[5, t ] + V[6, t] + V[7, t ] + V[8, t] + V[9, t ] + V[10, t]
+V[11, t ] + V[12, t] + V[13, t ] + V[14, t] + V[15, t ] + V[16, t] + V[17, t ];
```

```
In[∘]:= a0 = 15/512; (* Der Wert ist unten ausführlich bestimmt worden. Hier schon genommen,
damit die Zeichnung klappt*)
```

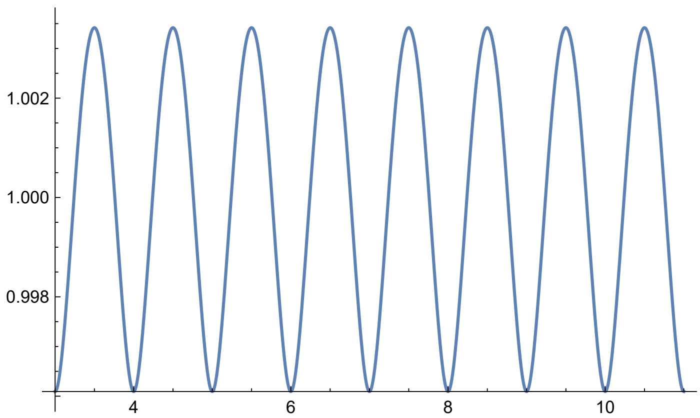
```
In[∘]:= Plot[{V[0, t ], V[1, t], V[2, t ], V[3, t ],
        [stelle Funktion graphisch dar
  V[4, t], V[5, t ], V[6, t], V[7, t ], V[8, t], V[9, t ] + V[10, t]
   + V[11, t ] + V[12, t] + V[13, t ] + V[14, t] + V[15, t ] + V[16, t] + V[17, t ]},
 {t, 0, 21}, PlotRange → All] (*V0 bis V8 einzeln, V9 bis V17 nur Summe*)
                 [Koordinatenb··· [alle
```

Out[∘]=


Es zeigt sich, dass sie Summe (hier von V9 bis V17) nicht!!!! exakt 1 ist.

```
In[∘]:= a0 = 15/512; Plot[{su[t]}, {t, 3, 11} (*,PlotRange→{0,1.2}*)]
                     [stelle Funktion graphisch dar      [Koordinatenbereich der Graph

a0 =.
```

Out[∘]=


Die Summe schwankt im Wirkungsbereich gleichmäßig etwa um 1+/- 0.004

○ Bedingung zur Bestimmung von a0, Methode 1

Das ist eine Periode der blauen Welle:

In[∘]:=
```
swerte = Table[su[t], {t, 5, 6, 1/10}]
```
Tabelle

Out[∘]= $\left\{34\, a0, \frac{85081\, a0}{2500}, \frac{21314\, a0}{625}, \frac{85441\, a0}{2500}, \frac{21394\, a0}{625}, \frac{137\, a0}{4}, \frac{21394\, a0}{625}, \frac{85441\, a0}{2500}, \frac{21314\, a0}{625}, \frac{85081\, a0}{2500}, 34\, a0\right\}$

```
Integrate[su[t], {t, 5, 6}] (*der Funktionsmittelwert einer Periode*)
```
integriere

Out[∘]= $\frac{512\, a0}{15}$

Da alle relevanten Perioden gleich sind, ist das auch der Mittel aller relevanten Perioden. Also ist a0 = $\frac{15}{512}$ ein vernünftiger Wert.

○ Bedingung zur Bestimmung von a0 Methode 2

○ Bedingung zur Bestimmung von a0, Methode ursprünglich

Eine andere Idee ist es, den stets gleichen Maximalwert der Schwankung auf 1 zu setzen.

In[∘]:=
```
a0 =.; V[i_, t_] := V[i - 1, t - 1]; sum[t_] := Sum[V[i, t], {i, 0, 24}]
```
summiere

```
Solve[sum[5 + 1/2] == 1, a0]
```
löse

Out[∘]= $\left\{\left\{a0 \to \frac{4}{137}\right\}\right\}$

In[∘]:=
```
a0 = a0 /. a0 → 4/137 // N
```
n

Out[∘]= 0.0291971

In[∘]:=
```
swerte
```

Out[∘]= {0.992701, 0.993647, 0.995691, 0.997851, 0.999428, 1., 0.999428, 0.997851, 0.995691, 0.993647, 0.992701}

Allerdings schwankt hier die Summe in etwa [0, 9927, 1] regelmäßig .

■ Wirkliche B-Splines

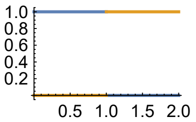
In[∘]:=
```
B[0, 0, t_] := If[0 ≤ t && t ≤ 1 , 1, 0]
```
wenn

```
B[1, 0, t_] := B[0, 0, t - 1]
```

In[∘]:=
```
B[1, 0, t]
```

Out[∘]= If[0 ≤ -1 + t && -1 + t ≤ 1, 1, 0]

In[∘]:= Plot[{B[0, 0, t], B[1, 0, t]}, {t, 0, 2}]
stelle Funktion graphisch dar

Out[∘]=


In[∘]:= p1 = 1;

In[∘]:= B[0, 1, t_] := t B[0, 0, t] + (2 - t) B[1, 0, t]

In[∘]:= B[0, 1, t]

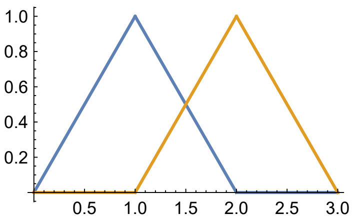
Out[∘]= (2 - t) If[0 ≤ -1 + t && -1 + t ≤ 1, 1, 0] + t If[0 ≤ t && t ≤ 1, 1, 0]

In[∘]:= B[1, 1, t_] := B[0, 1, t - 1]

In[∘]:= B[1, 1, t]

Out[∘]= (3 - t) If[0 ≤ -2 + t && -2 + t ≤ 1, 1, 0] + (-1 + t) If[0 ≤ -1 + t && -1 + t ≤ 1, 1, 0]

Plot[{B[0, 1, t], B[1, 1, t]}, {t, 0, 3}]
stelle Funktion graphisch dar

Out[∘]=


p2 = 2;

B[0, 2, t_] := $\frac{t}{p2}$ B[0, 1, t] + $\frac{p2 + 1 - t}{p2}$ B[1, 1, t]

In[∘]:= B[0, 2, t]

Out[∘]= $\frac{1}{2}$ (3 - t) ((3 - t) If[0 ≤ -2 + t && -2 + t ≤ 1, 1, 0] + (-1 + t) If[0 ≤ -1 + t && -1 + t ≤ 1, 1, 0]) +
$\frac{1}{2}$ t ((2 - t) If[0 ≤ -1 + t && -1 + t ≤ 1, 1, 0] + t If[0 ≤ t && t ≤ 1, 1, 0])

In[∘]:= B[1, 2, t_] := B[0, 2, t - 1]

In[∘]:= B[1, 2, t]

Out[∘]= $\frac{1}{2}$ (4 - t) ((4 - t) If[0 ≤ -3 + t && -3 + t ≤ 1, 1, 0] + (-2 + t) If[0 ≤ -2 + t && -2 + t ≤ 1, 1, 0]) +
$\frac{1}{2}$ (-1 + t)
((3 - t) If[0 ≤ -2 + t && -2 + t ≤ 1, 1, 0] + (-1 + t) If[0 ≤ -1 + t && -1 + t ≤ 1, 1, 0])

In[∘]:= B[2, 2, t_] := B[0, 2, t - 2]

In[∘]:=
```
B[2, 2, t]
```

Out[∘]=

$$\frac{1}{2}(5-t)\,((5-t)\,\text{If}[0\le -4+t\,\&\&\,-4+t\le 1,1,0]+(-3+t)\,\text{If}[0\le -3+t\,\&\&\,-3+t\le 1,1,0])+\frac{1}{2}(-2+t)\,((4-t)\,\text{If}[0\le -3+t\,\&\&\,-3+t\le 1,1,0]+(-2+t)\,\text{If}[0\le -2+t\,\&\&\,-2+t\le 1,1,0])$$

Meins, unten blau, passt

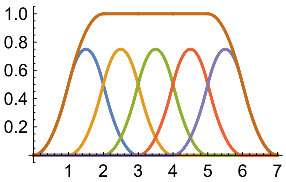
In[∘]:=

$$\text{BB}[0, 2, t\_] := \text{If}\left[0\le t\le 1, \frac{t^2}{2}, \text{If}\left[1\le t\,\&\&\,t\le 2, -t^2+3t-\frac{3}{2}, \text{If}\left[2\le t\le 3, \frac{(3-t)^2}{2}, 0\right]\right]\right]$$

wenn wenn wenn

In[∘]:=
```
Plot[{B[0, 2, t](*,BB[0,2,t]*),
  B[1, 2, t], B[0, 2, t - 2], B[0, 2, t - 3], B[0, 2, t - 4],
  B[0, 2, t] + B[1, 2, t] + B[0, 2, t - 2] + B[0, 2, t - 3] + B[0, 2, t - 4]}, {t, 0, 7}]
```
stelle Funktion graphisch dar

Out[∘]=



Von diesen Hügeln erhält man beliebig viele durch Verschieben.
Bei n Hügeln verwendbar von 2 bis n, Bereich 0 bis n+2, bei p=2

○ p3=3

```
p3 = 3;
```

$$\text{B}[0, 3, t\_] := \frac{t}{\text{p3}}\,\text{B}[0, 2, t] + \frac{\text{p3}+1-t}{\text{p3}}\,\text{B}[1, 2, t]$$

In[∘]:=
```
B[0, 3, t]
```

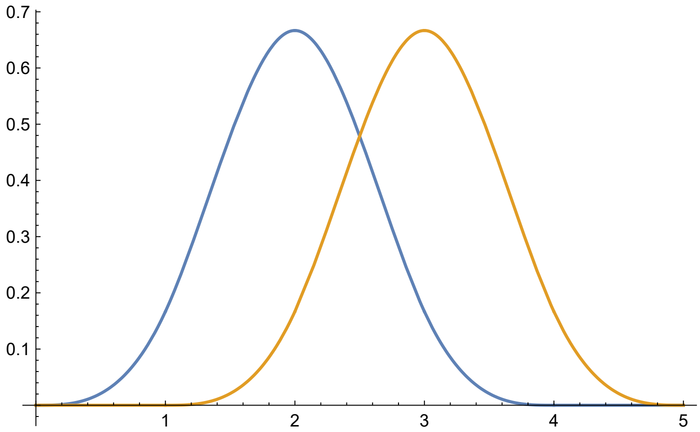
Out[∘]=

$$\frac{1}{3}(4-t)\left(\frac{1}{2}(4-t)\,((4-t)\,\text{If}[0\le -3+t\,\&\&\,-3+t\le 1,1,0]+(-2+t)\,\text{If}[0\le -2+t\,\&\&\,-2+t\le 1,1,0])+\frac{1}{2}(-1+t)\,((3-t)\,\text{If}[0\le -2+t\,\&\&\,-2+t\le 1,1,0]+(-1+t)\,\text{If}[0\le -1+t\,\&\&\,-1+t\le 1,1,0])\right)+\frac{1}{3}t\left(\frac{1}{2}(3-t)\,((3-t)\,\text{If}[0\le -2+t\,\&\&\,-2+t\le 1,1,0]+(-1+t)\,\text{If}[0\le -1+t\,\&\&\,-1+t\le 1,1,0])+\frac{1}{2}t\,((2-t)\,\text{If}[0\le -1+t\,\&\&\,-1+t\le 1,1,0]+t\,\text{If}[0\le t\,\&\&\,t\le 1,1,0])\right)$$

In[∘]:=
```
B[1, 3, t_] := B[0, 3, t - 1];
```

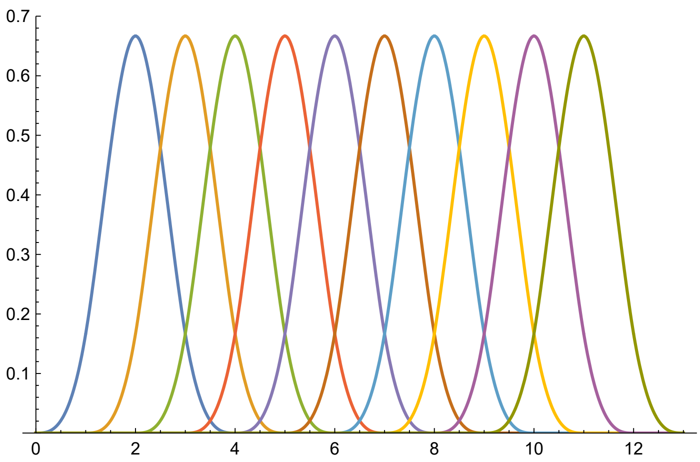
In[∘]:=
```
Plot[{B[0, 3, t], B[1, 3, t]}, {t, 0, 5}]
```
stelle Funktion graphisch dar

Out[∘]=



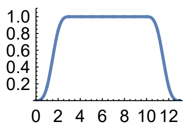
In[∘]:=
```
alle = Plot[{B[0, 3, t], B[0, 3, t - 1], B[0, 3, t - 2], B[0, 3, t - 3]
  , B[0, 3, t - 4], B[0, 3, t - 5], B[0, 3, t - 6], B[0, 3, t - 7]
  , B[0, 3, t - 8], B[0, 3, t - 9]}, {t, 0, 13}, PlotRange → {0, 0.7}]
```
stelle Funktion graphisch dar
Koordinatenbereich der Grapl

Out[∘]=



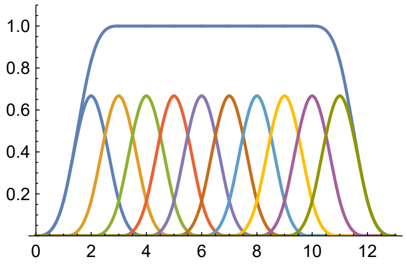
In[∘]:=
```
Summe = Plot[Sum[B[0, 3, t - i], {i, 0, 9}], {t, 0, 13}, PlotRange → {0, 1.1}]
```
stell··· summiere
Koordinatenbereich der Grapl

Out[∘]=



In[∘]:=
```
Show[Summe, alle]
```
zeige an

Out[∘]=



Nicht-uniforme B-Splines

Knotenliste

```
In[∘]:= T = {0, 1/2, 3/4, 6/5, 2, 5/2, 3, 17/5, 18/5}

In[∘]:= {0, 1/2, 3/4, 6/5, 2, 5/2, 3, 17/5, 18/5}

Out[∘]= {0, 1/2, 3/4, 6/5, 2, 5/2, 3, 17/5, 18/5}

In[∘]:= tt[i_] := T[[i]]; tt[3] × B[i, 3, t]

Out[∘]= 3/4 B[i, 3, t]

In[∘]:= Table[tt[i], {i, 1, 9}]
        Tabelle

Out[∘]= {0, 1/2, 3/4, 6/5, 2, 5/2, 3, 17/5, 18/5}
```

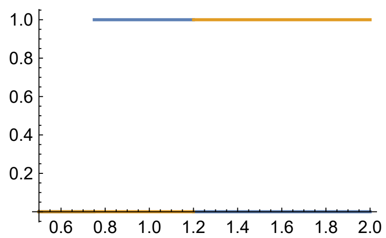
Start

```
In[∘]:= BB[i_, 0, t_] := If[tt[i] ≤ t && t ≤ tt[i + 1] , 1, 0];
                          wenn
        BB[3, 0, t]

Out[∘]= If[3/4 ≤ t && t ≤ 6/5, 1, 0]

In[∘]:= Plot[{BB[3, 0, t], BB[4, 0, t]}, {t, 1/2, 2}]
        stelle Funktion graphisch dar
```

Out[∘]=



Rekursion

```
In[∘]:= p =.

In[∘]:= BB[i_, p_, t_] := (t - tt[i])/(tt[i + p] - tt[i]) BB[i, p - 1, t] + (tt[i + p + 1] - t)/(tt[i + p + 1] - tt[i + 1]) BB[i + 1, p - 1, t]
```

● p=1 entsteht

In[∘]:= `BB[3, 1, t]`

Out[∘]= $\frac{20}{9}\left(-\frac{3}{4}+t\right)\text{If}\left[\frac{3}{4}\le t\,\&\&\,t\le\frac{6}{5},1,0\right]+\frac{5}{4}(2-t)\,\text{If}\left[\frac{6}{5}\le t\,\&\&\,t\le 2,1,0\right]$

In[∘]:= `BB[4, 1, t]`

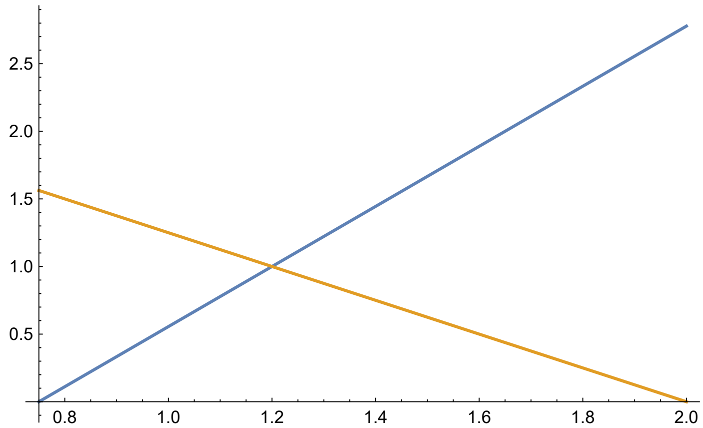
Out[∘]= $\frac{5}{4}\left(-\frac{6}{5}+t\right)\text{If}\left[\frac{6}{5}\le t\,\&\&\,t\le 2,1,0\right]+2\left(\frac{5}{2}-t\right)\text{If}\left[2\le t\,\&\&\,t\le\frac{5}{2},1,0\right]$

In[∘]:= ⋮

Out[∘]= ⋮

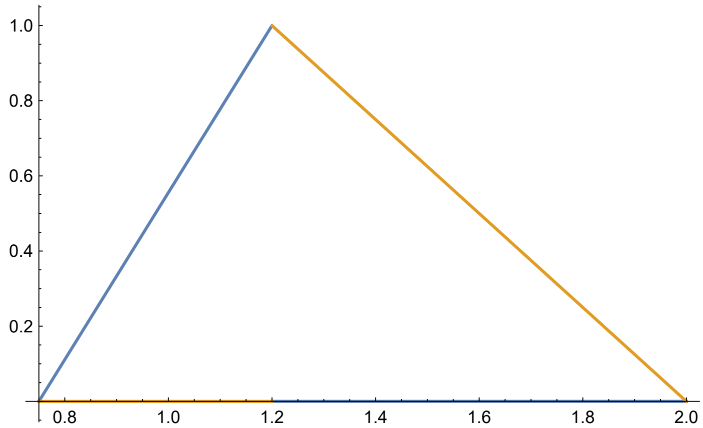
In[∘]:= $\text{Plot}\left[\left\{\frac{20}{9}\left(-\frac{3}{4}+t\right),\frac{5}{4}(2-t)\right\},\{t,\text{tt}[3],\text{tt}[5]\}\right]$
stelle Funktion graphisch dar

Out[∘]=



In[∘]:= $\text{Plot}\left[\left\{\frac{20}{9}\left(-\frac{3}{4}+t\right)\text{BB}[3,0,t],\frac{5}{4}(2-t)\,\text{BB}[4,0,t]\right\},\{t,\text{tt}[3],\text{tt}[5]\}\right]$
stelle Funktion graphisch dar

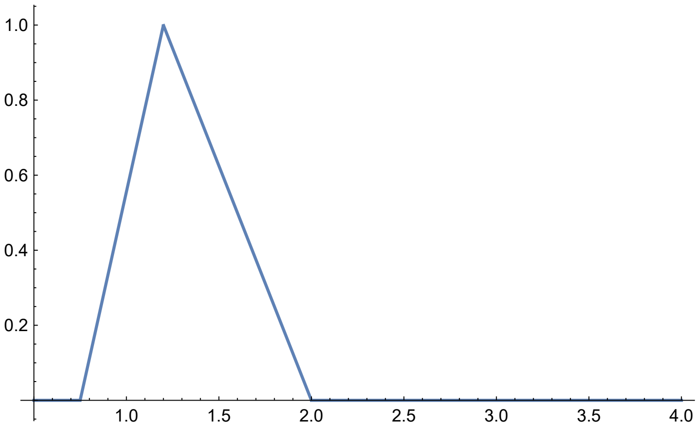
Out[∘]=



In[ ]:= Plot$\left[\left\{\frac{20}{9}\left(-\frac{3}{4}+t\right)\text{BB}[3, 0, t] + \frac{5}{4}(2-t)\,\text{BB}[4, 0, t]\right\}, \left\{t, \frac{1}{2}, 4\right\}\right]$

stelle Funktion graphisch dar

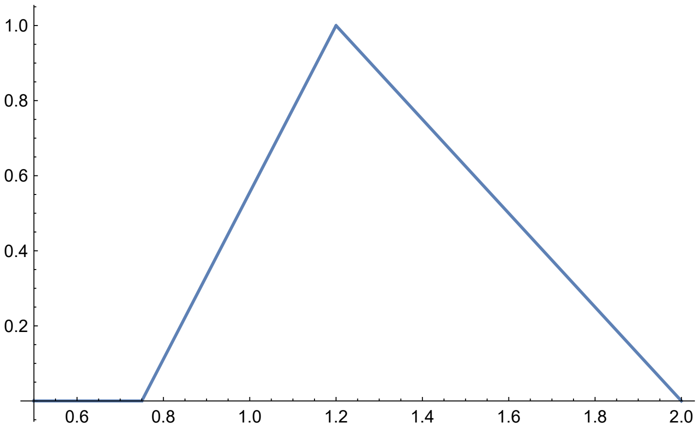
Out[ ]=



In[ ]:= Plot$\left[\{\text{BB}[3, 1, t] \;//\; \text{Evaluate}\}, \left\{t, \frac{1}{2}, 2\right\}\right]$

stelle Funktion graphisch ⋯ werte aus

Out[ ]=



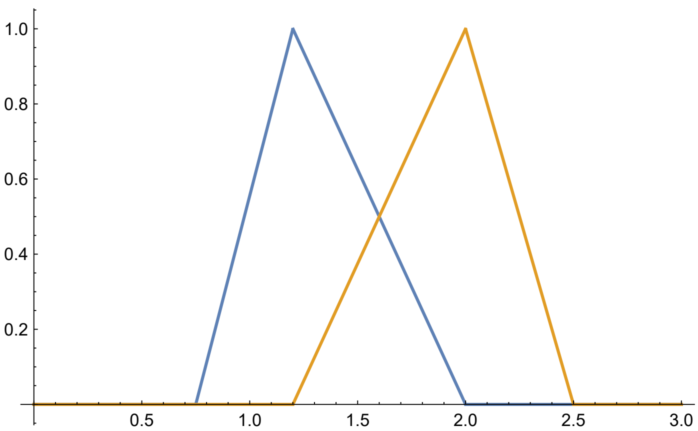
In[ ]:= BB[4, 1, t]

Out[ ]= $\frac{5}{4}\left(-\frac{6}{5}+t\right)\text{If}\left[\frac{6}{5} \le t \,\&\&\, t \le 2, 1, 0\right] + 2\left(\frac{5}{2}-t\right)\text{If}\left[2 \le t \,\&\&\, t \le \frac{5}{2}, 1, 0\right]$

In[ ]:= Plot[{BB[3, 1, t], BB[4, 1, t]}, {t, 0, 3}]

stelle Funktion graphisch dar

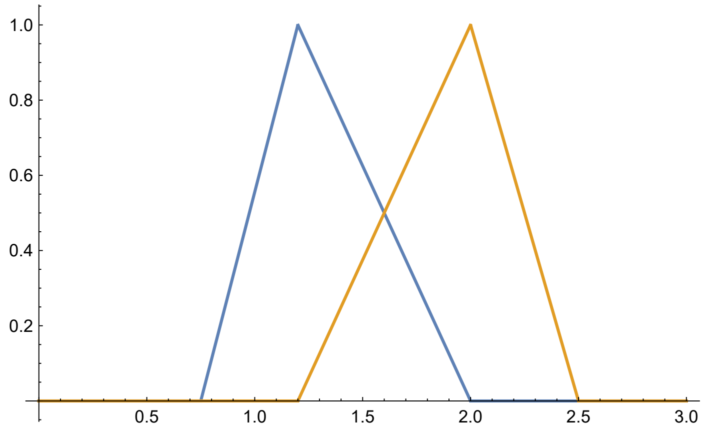
Out[ ]=



p=1 fertig

In[ ]:=
```
Plot[{BB[3, 1, t], BB[4, 1, t]}, {t, 0, 3}]
```
stelle Funktion graphisch dar

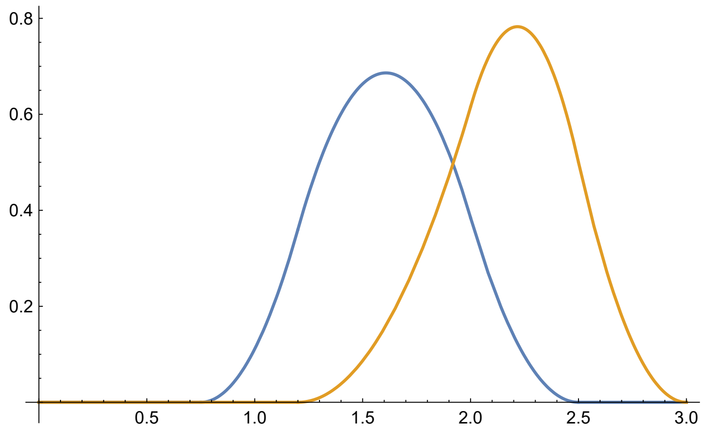
Out[ ]=



p=2 fertig

In[ ]:=
```
Plot[{BB[3, 2, t], BB[4, 2, t]}, {t, 0, 3}]
```
stelle Funktion graphisch dar

Out[ ]=



p=3 , eine Fkt fertig

In[ ]:=
```
nr34 = Table[BB[i, 3, t], {i, 3, 4}]
```
Tabelle

Out[ ]=

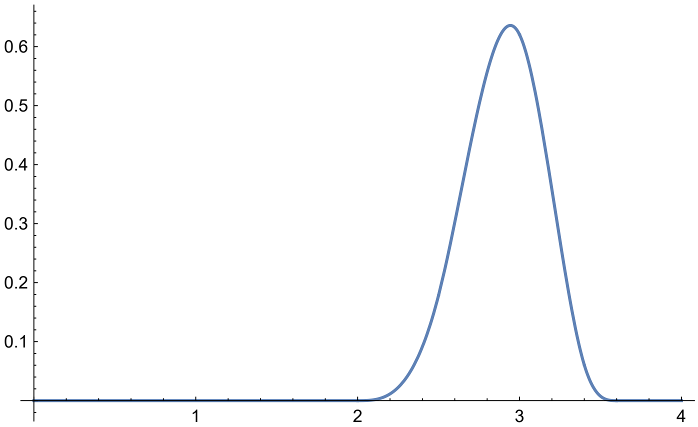
$$
\begin{aligned}
&\Big\{\tfrac{4}{7}\left(-\tfrac{3}{4}+t\right)\\
&\quad\Big(\tfrac{4}{5}\left(-\tfrac{3}{4}+t\right)\Big(\tfrac{20}{9}\left(-\tfrac{3}{4}+t\right)\text{If}\big[\tfrac{3}{4}\le t\,\&\&\,t\le\tfrac{6}{5},1,0\big]+\tfrac{5}{4}(2-t)\,\text{If}\big[\tfrac{6}{5}\le t\,\&\&\,t\le 2,1,0\big]\Big)+\\
&\qquad\tfrac{10}{13}\left(\tfrac{5}{2}-t\right)\Big(\tfrac{5}{4}\left(-\tfrac{6}{5}+t\right)\text{If}\big[\tfrac{6}{5}\le t\,\&\&\,t\le 2,1,0\big]+2\left(\tfrac{5}{2}-t\right)\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]\Big)\Big)+\\
&\quad\tfrac{5}{9}(3-t)\Big(\tfrac{10}{13}\left(-\tfrac{6}{5}+t\right)\Big(\tfrac{5}{4}\left(-\tfrac{6}{5}+t\right)\text{If}\big[\tfrac{6}{5}\le t\,\&\&\,t\le 2,1,0\big]+\\
&\qquad\qquad 2\left(\tfrac{5}{2}-t\right)\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]\Big)+(3-t)\\
&\qquad\quad\Big(2(-2+t)\,\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]+2(3-t)\,\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]\Big)\Big)\Big),\ \tfrac{5}{9}\left(-\tfrac{6}{5}+t\right)\\
&\quad\Big(\tfrac{10}{13}\left(-\tfrac{6}{5}+t\right)\Big(\tfrac{5}{4}\left(-\tfrac{6}{5}+t\right)\text{If}\big[\tfrac{6}{5}\le t\,\&\&\,t\le 2,1,0\big]+2\left(\tfrac{5}{2}-t\right)\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]\Big)+\\
&\qquad(3-t)\Big(2(-2+t)\,\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]+2(3-t)\,\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]\Big)\Big)+\\
&\quad\tfrac{5}{7}\left(\tfrac{17}{5}-t\right)\Big((-2+t)\Big(2(-2+t)\,\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]+2(3-t)\,\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]\Big)+\\
&\qquad\tfrac{10}{9}\left(\tfrac{17}{5}-t\right)\Big(2\left(-\tfrac{5}{2}+t\right)\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]+\tfrac{5}{2}\left(\tfrac{17}{5}-t\right)\text{If}\big[3\le t\,\&\&\,t\le\tfrac{17}{5},1,0\big]\Big)\Big)\Big\}
\end{aligned}
$$

In[ ]:=
```
BB[5, 3, t]
```

Out[ ]=

$$
\begin{aligned}
&\tfrac{5}{7}(-2+t)\Big((-2+t)\Big(2(-2+t)\,\text{If}\big[2\le t\,\&\&\,t\le\tfrac{5}{2},1,0\big]+2(3-t)\,\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]\Big)+\\
&\quad\tfrac{10}{9}\left(\tfrac{17}{5}-t\right)\Big(2\left(-\tfrac{5}{2}+t\right)\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]+\tfrac{5}{2}\left(\tfrac{17}{5}-t\right)\text{If}\big[3\le t\,\&\&\,t\le\tfrac{17}{5},1,0\big]\Big)\Big)+\\
&\tfrac{10}{11}\left(\tfrac{18}{5}-t\right)\Big(\tfrac{10}{9}\left(-\tfrac{5}{2}+t\right)\Big(2\left(-\tfrac{5}{2}+t\right)\text{If}\big[\tfrac{5}{2}\le t\,\&\&\,t\le 3,1,0\big]+\\
&\qquad\tfrac{5}{2}\left(\tfrac{17}{5}-t\right)\text{If}\big[3\le t\,\&\&\,t\le\tfrac{17}{5},1,0\big]\Big)+\\
&\quad\tfrac{5}{3}\left(\tfrac{18}{5}-t\right)\Big(\tfrac{5}{2}(-3+t)\,\text{If}\big[3\le t\,\&\&\,t\le\tfrac{17}{5},1,0\big]+5\left(\tfrac{18}{5}-t\right)\text{If}\big[\tfrac{17}{5}\le t\,\&\&\,t\le\tfrac{18}{5},1,0\big]\Big)\Big)
\end{aligned}
$$

In[ ]:=
```
Plot[BB[5, 3, t], {t, 0, 4}]
```
stelle Funktion graphisch dar

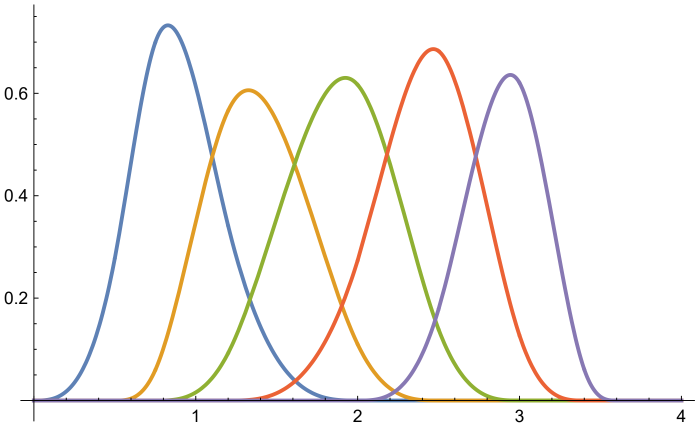
Out[ ]=


In[ ]:=
```
Table[BB[i, 3, t], {i, 1, 5}];
```
Tabelle

p=3 alle fertig

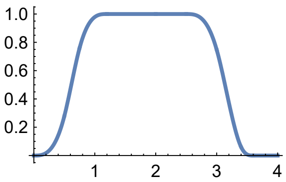
In[ ]:=
```
Table[BB[i, 3, t], {i, 1, 5}];
```
Tabelle

In[ ]:=
```
Plot[{BB[1, 3, t], BB[2, 3, t], BB[3, 3, t], BB[4, 3, t], BB[5, 3, t]},
  {t, 0, 4}, PlotStyle → Thick]
```
stelle Funktion graphisch dar; Darstellungsstil; dick

Out[ ]=


Dies ist Abb. 5.23 a) in unserem Buch.

In[ ]:=
```
Plot[{BB[1, 3, t] + BB[2, 3, t] + BB[3, 3, t] + BB[4, 3, t] + BB[5, 3, t]},
  {t, 0, 4}, PlotStyle → Thick]
```
stelle Funktion graphisch dar; Darstellungsstil; dick

Out[ ]=


Rationale Béziersplines

Ausführliche Dateien für Kreis und Trisektrix (das sind Links)

- Bezier Splines
- Gewichte für den Kreis und Trisektrix (Nenner = $1 + t^2$)

Versuch für die Gerono-Lemniskate

Nicht-uniforme didaktische B-Splines