Lineare partielle Differentialgleichungen 2. Ordnung

Elliptisch: Laplace auf Einheitskreis

In[1]:=
$$\text{Simplify}\left[\int_0^{2\pi}\frac{\frac{1}{2}\,\text{Sin}[3\,t]+\frac{1}{8}\,\text{Cos}[17\,t]}{(x-\text{Cos}[t])^2+(y-\text{Sin}[t])^2}\,dt,\ \text{Abs}[x+i\,y]<1\right]$$

Out[1]=
$$\begin{cases}
\frac{1}{8}\left(\frac{4\,i\,\pi\,(1+x^4+y^2+y^4+x^2\,(1+2\,y^2))}{(x-i\,y)^3}-\frac{1}{(x-i\,y)^{17}}\pi\,\big(1+x^{32}+y^2+y^4+y^6+y^8+y^{10}+y^{12}+y^{14}+y^{16}+y^{18}+y^{20}+y^{22}+y^{24}+y^{26}+y^{28}+y^{30}+y^{32}+x^{30}\,(1+16\,y^2)+x^{28}\,(1+15\,y^2+120\,y^4)+x^{26}\,(1+14\,y^2+105\,y^4+560\,y^6)+x^{24}\,(1+13\,y^2+91\,y^4+455\,y^6+1820\,y^8)+x^{22}\,(1+12\,y^2+78\,y^4+364\,y^6+1365\,y^8+4368\,y^{10})+x^{20}\,(1+11\,y^2+66\,y^4+286\,y^6+1001\,y^8+3003\,y^{10}+8008\,y^{12})+x^{18}\,(1+10\,y^2+55\,y^4+220\,y^6+715\,y^8+2002\,y^{10}+5005\,y^{12}+11\,440\,y^{14})+x^{16}\,(1+9\,y^2+45\,y^4+165\,y^6+495\,y^8+1287\,y^{10}+3003\,y^{12}+6435\,y^{14}+12\,870\,y^{16})+x^{14}\,(1+8\,y^2+36\,y^4+120\,y^6+330\,y^8+792\,y^{10}+1716\,y^{12}+3432\,y^{14}+6435\,y^{16}+11\,440\,y^{18})+x^{12}\,(1+7\,y^2+28\,y^4+84\,y^6+210\,y^8+462\,y^{10}+924\,y^{12}+1716\,y^{14}+3003\,y^{16}+5005\,y^{18}+8008\,y^{20})+x^{10}\,(1+6\,y^2+21\,y^4+56\,y^6+126\,y^8+252\,y^{10}+462\,y^{12}+792\,y^{14}+1287\,y^{16}+2002\,y^{18}+3003\,y^{20}+4368\,y^{22})+x^8\,(1+5\,y^2+15\,y^4+35\,y^6+70\,y^8+126\,y^{10}+210\,y^{12}+330\,y^{14}+495\,y^{16}+715\,y^{18}+1001\,y^{20}+1365\,y^{22}+1820\,y^{24})+x^6\,(1+4\,y^2+10\,y^4+20\,y^6+35\,y^8+56\,y^{10}+84\,y^{12}+120\,y^{14}+165\,y^{16}+220\,y^{18}+286\,y^{20}+364\,y^{22}+455\,y^{24}+560\,y^{26})+x^4\,(1+3\,y^2+6\,y^4+10\,y^6+15\,y^8+21\,y^{10}+28\,y^{12}+36\,y^{14}+45\,y^{16}+55\,y^{18}+66\,y^{20}+78\,y^{22}+91\,y^{24}+105\,y^{26}+120\,y^{28})+x^2\,(1+2\,y^2+3\,y^4+4\,y^6+5\,y^8+6\,y^{10}+7\,y^{12}+8\,y^{14}+9\,y^{16}+10\,y^{18}+11\,y^{20}+12\,y^{22}+13\,y^{24}+14\,y^{26}+15\,y^{28}+16\,y^{30})\big)\right) & \frac{1}{\text{Abs}[x-i\,y]}<1 \\
-\frac{1}{4\,(-1+x^2+y^2)}\pi\,(x^{17}+12\,x^2\,y-136\,x^{15}\,y^2-4\,y^3+2380\,x^{13}\,y^4-12\,376\,x^{11}\,y^6+24\,310\,x^9\,y^8-19\,448\,x^7\,y^{10}+6188\,x^5\,y^{12}-680\,x^3\,y^{14}+17\,x\,y^{16}) & \text{True}
\end{cases}$$

In[2]:=
$$\text{uu}[x\_,y\_]:=\frac{1-x^2-y^2}{2\,\pi}\left(-\frac{1}{4\,(-1+x^2+y^2)}\pi\,(x^{17}+12\,x^2\,y-136\,x^{15}\,y^2-4\,y^3+2380\,x^{13}\,y^4-12\,376\,x^{11}\,y^6+24\,310\,x^9\,y^8-19\,448\,x^7\,y^{10}+6188\,x^5\,y^{12}-680\,x^3\,y^{14}+17\,x\,y^{16})\right)$$

In[3]:=
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uu[x, y] // Simplify
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Out[3]= $\frac{1}{8}\left(x^{17}+12 x^2 y-136 x^{15} y^2-4 y^3+2380 x^{13} y^4-12376 x^{11} y^6+24310 x^9 y^8-19448 x^7 y^{10}+6188 x^5 y^{12}-680 x^3 y^{14}+17 x y^{16}\right)$

In[4]:=
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uu[.9, .9]
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Out[4]= 6.0657

In[5]:=
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Plot3D[uu[x, y], {x, -1, 1}, {y, -1, 1}, RegionFunction → Function[{x, y}, x^2 + y^2 < 1],
 BoxRatios → Automatic, AxesLabel → Automatic]
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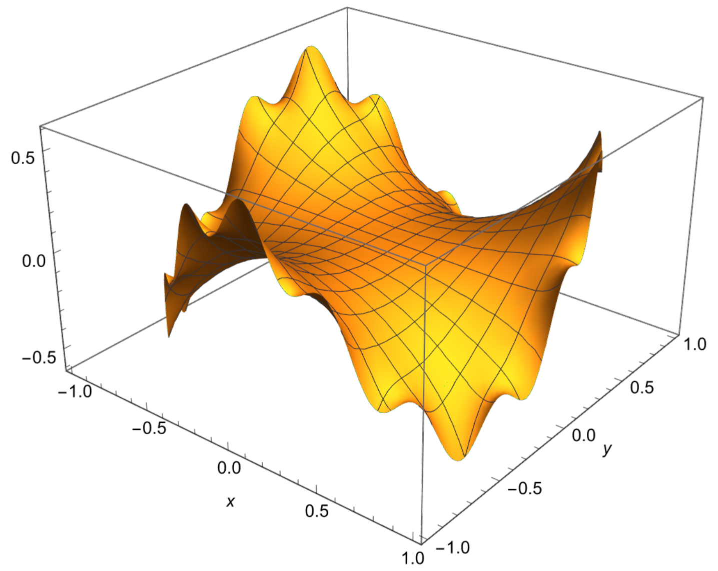
Out[5]=



In[6]:=
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ParametricPlot3D[{{Cos[t], Sin[t], 1/2 Sin[3 t] + 1/8 Cos[17 t]}, {Cos[t], Sin[t], 0}},
 {t, 0, 2 π}, BoxRatios → Automatic, AxesLabel → {"x", "y", "z"}]
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Out[6]=