Phasenraumbilder zu nichtlineare DGl-Systemen

Buch:Höhere Mathematik sehen und verstehen, Haftendorn, Riebesehl, Dammer, Springer Spektrum, Feb. 2021

Datei DGlSys-nonlinear-Bilder.nb zu Abschnitt 4.6.6 Seite 342, Abb. 4.27

Bilder zu Lösungskurven, instabiler Fixpunkt und instabiler Knoten

```
eqs = {x'[t] == 3/10 x[t]^2 - 4/10 y[t], y'[t] == -2/10 x[t] + 12/10 y[t]^2};
```

```
eqs /. {x'[t] → x₁', y'[t] → x₂', x[t] → x₁, y[t] → x₂} // TeXForm
```

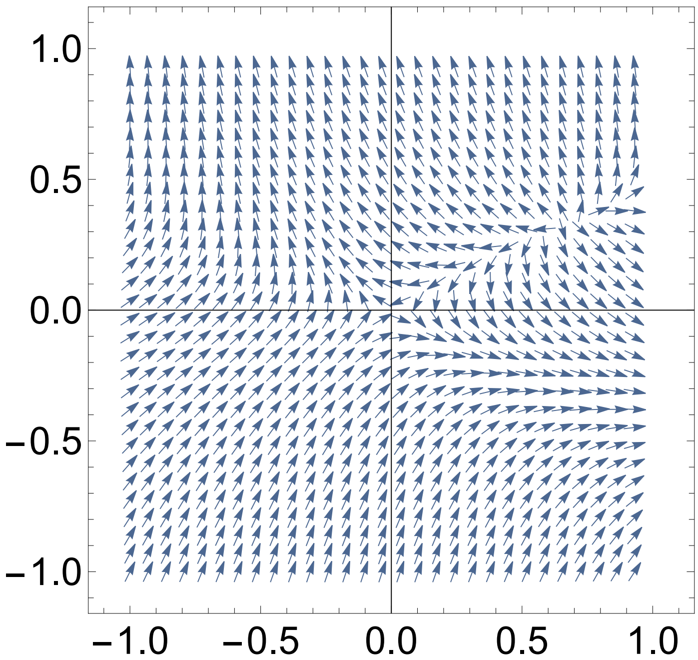
```
\left\{x_1'=\frac{3 x_1^2}{10}-\frac{2
   x_2}{5},x_2'=\frac{6
   x_2^2}{5}-\frac{x_1}{5}\right\}
```

```
solpts = Solve[eqs /. {x'[t] → 0, y'[t] → 0, x[t] → x, y[t] → y}][[{1, 2}]]
```

$\{\{x \to 0, y \to 0\}, \{x \to \frac{2}{3}, y \to \frac{1}{3}\}\}$

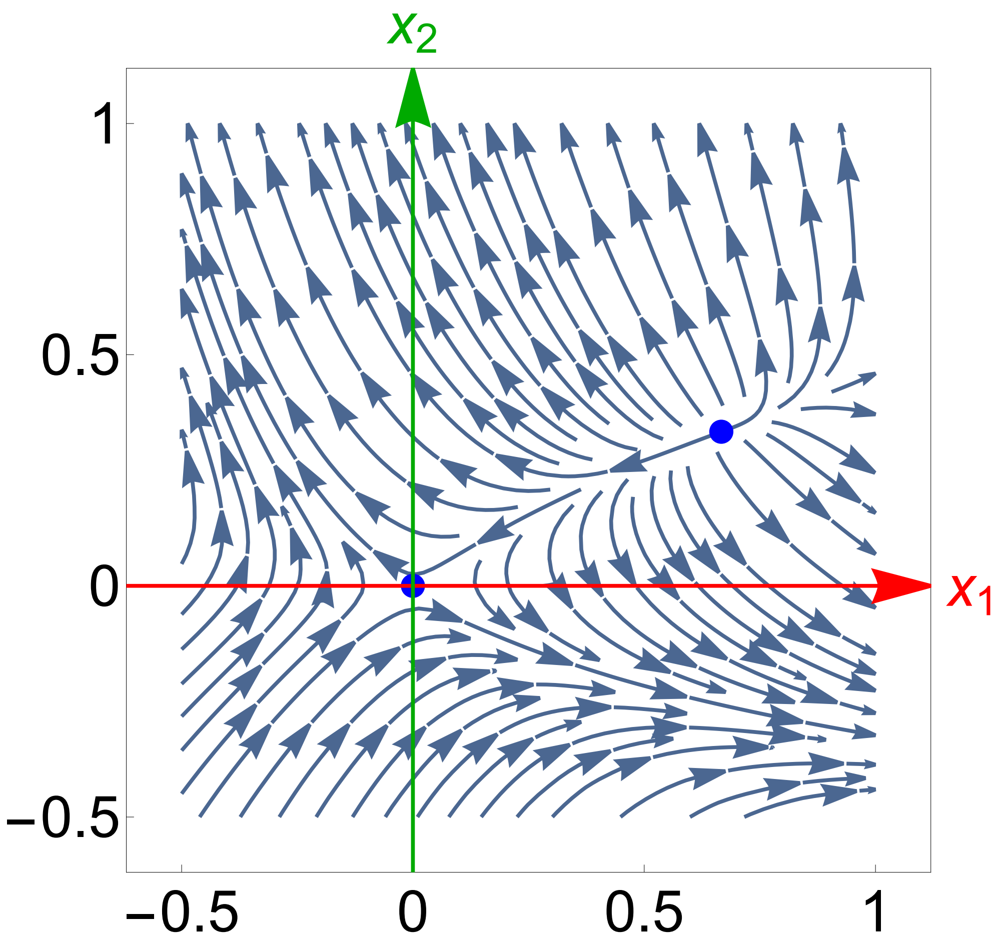
```
pts = Graphics[{Blue, PointSize[0.03], Point[{x, y} /. solpts]}];
```

```
vp = VectorPlot[Transpose[List @@ # & /@ eqs][[2]] /. {x[t] → x, y[t] → y},
  {x, -1, 1}, {y, -1, 1}, VectorScale → {0.03, Automatic, None}, Axes → True,
  GridLines → None, BaseStyle → FontSize → 20, VectorPoints → 30]
```



```
stream = StreamPlot[Transpose[List @@ # & /@ eqs][[2]] /. {x[t] → x, y[t] → y},
    {x, -0.5, 1}, {y, -0.5, 1}, Axes → True, GridLines → None, BaseStyle → FontSize → 30,
    FrameTicks → {{{-0.5, 0, 0.5, 1}, None}, {{-0.5, 0, 0.5, 1}, None}}, AxesStyle →
     {{Red, Thick, Arrowheads[0.07]}, {Green // Darker, Thick, Arrowheads[0.07]}},
    AxesLabel → {"x1", "x2"}, StreamStyle -> Thick, StreamScale -> Large];
```

```
Show[stream, pts]
```



- Bilder zu Lösungskurven, diesmal mit Wirbel

  - allgemein Suche nach Wirbel

```
f[x_, y_] := a x^2 + b y; g[x_, y_] := c y^2 + d x;
```

$$J = \begin{pmatrix} \partial_x f[x, y] & \partial_y f[x, y] \\ \partial_x g[x, y] & \partial_y g[x, y] \end{pmatrix}$$

{{2 a x, b}, {d, 2 c y}}

```
eig = Eigenvalues[J]
```

$$\left\{a x + c y - \sqrt{b d + a^2 x^2 - 2 a c x y + c^2 y^2},\ a x + c y + \sqrt{b d + a^2 x^2 - 2 a c x y + c^2 y^2}\right\}$$

```
fix = Solve[{f[x, y] == 0, g[x, y] == 0}, {x, y}]
```

$$\left\{\{x\to 0, y\to 0\}, \left\{x\to -\frac{b^{2/3} d^{1/3}}{a^{2/3} c^{1/3}}, y\to -\frac{b^{1/3} d^{2/3}}{a^{1/3} c^{2/3}}\right\}, \left\{x\to \frac{(-1)^{1/3} b^{2/3} d^{1/3}}{a^{2/3} c^{1/3}}, y\to -\frac{(-1)^{2/3} b^{1/3} d^{2/3}}{a^{1/3} c^{2/3}}\right\}, \left\{x\to -\frac{(-1)^{2/3} b^{2/3} d^{1/3}}{a^{2/3} c^{1/3}}, y\to \frac{(-1)^{1/3} b^{1/3} d^{2/3}}{a^{1/3} c^{2/3}}\right\}\right\}$$

```
fixeig = eig /. fix // Simplify
```

$$\left\{\left\{-\sqrt{b\,d}, \sqrt{b\,d}\right\}, \left\{-\frac{a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} - \frac{b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} - \sqrt{\frac{a^{2/3} b^{4/3} d^{2/3}}{c^{2/3}} - b\,d + \frac{b^{2/3} c^{2/3} d^{4/3}}{a^{2/3}}}, -\frac{a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} - \frac{b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} + \sqrt{\frac{a^{2/3} b^{4/3} d^{2/3}}{c^{2/3}} - b\,d + \frac{b^{2/3} c^{2/3} d^{4/3}}{a^{2/3}}}\right\},\right.$$

$$\left\{\frac{(-1)^{1/3} a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} - \frac{(-1)^{2/3} b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} - \sqrt{\frac{(-1)^{2/3} a^{2/3} b^{4/3} d^{2/3}}{c^{2/3}} - b\,d - \frac{(-1)^{1/3} b^{2/3} c^{2/3} d^{4/3}}{a^{2/3}}}, \frac{(-1)^{1/3} a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} - \frac{(-1)^{2/3} b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} + \sqrt{\frac{(-1)^{2/3} a^{2/3} b^{4/3} d^{2/3}}{c^{2/3}} - b\,d - \frac{(-1)^{1/3} b^{2/3} c^{2/3} d^{4/3}}{a^{2/3}}}\right\},$$

$$\left.\left\{-\frac{(-1)^{2/3} a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} + \frac{(-1)^{1/3} b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} - \sqrt{-\frac{(-1)^{1/3} a^{2/3} b^{4/3} d^{2/3}}{c^{2/3}} - b\,d + \frac{(-1)^{2/3} b^{2/3} c^{2/3} d^{4/3}}{a^{2/3}}}, -\frac{(-1)^{2/3} a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} + \frac{(-1)^{1/3} b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} + \sqrt{-\frac{(-1)^{1/3} a^{2/3} b^{4/3} d^{2/3}}{c^{2/3}} - b\,d + \frac{(-1)^{2/3} b^{2/3} c^{2/3} d^{4/3}}{a^{2/3}}}\right\}\right\}$$

$$\textbf{imeig = Solve}\left[-\frac{a^{1/3} b^{2/3} d^{1/3}}{c^{1/3}} - \frac{b^{1/3} c^{1/3} d^{2/3}}{a^{1/3}} == 0\right]$$

$$\left\{\{b\to 0\}, \{d\to 0\}, \left\{d\to -\frac{a^2 b}{c^2}\right\}\right\}$$

```
fixeig /. imeig[[3]] // PowerExpand // FullSimplify
```

$$\left\{\left\{-\frac{i\,a\,b}{c}, \frac{i\,a\,b}{c}\right\}, \left\{-\frac{i\sqrt{3}\,a\,b}{c}, -\frac{i\sqrt{3}\,a\,b}{c}\right\}, \left\{\frac{i\sqrt{3}\,a\,b}{c}, \frac{i\sqrt{3}\,a\,b}{c}\right\}, \left\{-\frac{\sqrt{3}\,a\,b}{c}, \frac{\sqrt{3}\,a\,b}{c}\right\}\right\}$$

○ nun eingesetzt

```
f[x_, y_] := a x^2 + b y; g[x_, y_] := c y^2 + d x /. imeig[[3]]
```

```
g[x, y]
```

$$-\frac{a^2 b x}{c^2} + c\,y^2$$

```
J = (∂x f[x, y]  ∂y f[x, y]
     ∂x g[x, y]  ∂y g[x, y])
```

$\{\{2\,a\,x, b\}, \{-\frac{a^2 b}{c^2}, 2\,c\,y\}\}$

```
eig = Eigenvalues[J]
```

$\{\frac{a\,c^2\,x + c^3\,y - \sqrt{-a^2\,b^2\,c^2 + a^2\,c^4\,x^2 - 2\,a\,c^5\,x\,y + c^6\,y^2}}{c^2}, \frac{a\,c^2\,x + c^3\,y + \sqrt{-a^2\,b^2\,c^2 + a^2\,c^4\,x^2 - 2\,a\,c^5\,x\,y + c^6\,y^2}}{c^2}\}$

```
fix = Solve[{f[x, y] == 0, g[x, y] == 0}, {x, y}]
```

$\{\{x \to 0, y \to 0\}, \{x \to \frac{b}{c}, y \to -\frac{a\,b}{c^2}\}, \{x \to \frac{(-1)^{2/3}\,b}{c}, y \to \frac{(-1)^{1/3}\,a\,b}{c^2}\}, \{x \to -\frac{(-1)^{1/3}\,b}{c}, y \to -\frac{(-1)^{2/3}\,a\,b}{c^2}\}\}$

```
fixeig = eig /. fix // PowerExpand // Simplify
```

$\{\{-\frac{i\,a\,b}{c}, \frac{i\,a\,b}{c}\}, \{-\frac{\sqrt{3}\,a\,b}{c}, \frac{\sqrt{3}\,a\,b}{c}\}, \{\frac{i\,\sqrt{3}\,a\,b}{c}, \frac{i\,\sqrt{3}\,a\,b}{c}\}, \{-\frac{i\,\sqrt{3}\,a\,b}{c}, -\frac{i\,\sqrt{3}\,a\,b}{c}\}\}$

○ Bilder dazu erzeugen

```
eqs = {x'[t] == x[t]^2 + y[t], y'[t] == -x[t] + y[t]^2};
```

```
solpts = Solve[eqs /. {x'[t] → 0, y'[t] → 0, x[t] → x, y[t] → y}][[{1, 2}]]
```

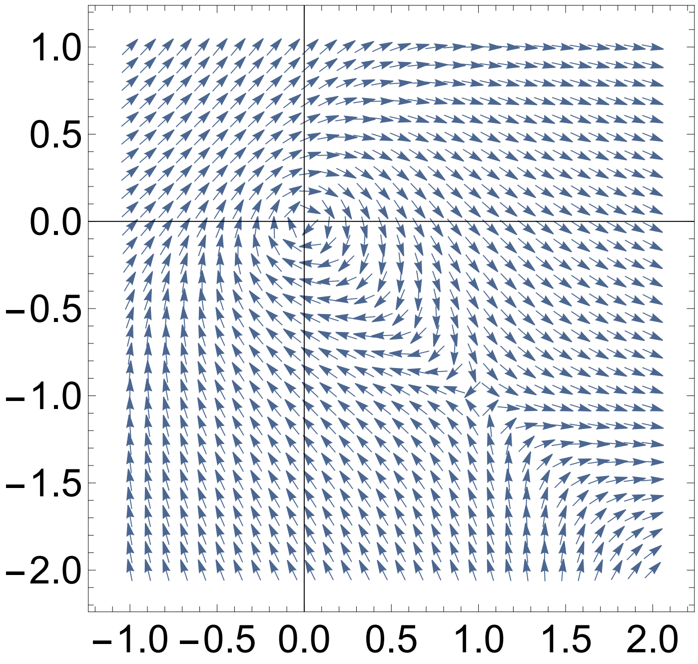
$\{\{x \to 0, y \to 0\}, \{x \to 1, y \to -1\}\}$

```
eqs /. {x'[t] → x1', y'[t] → x2', x[t] → x1, y[t] → x2} // TeXForm
```

```
\left\{x_1'=x_1^2+x_2,x_2'=x_2^2-x_1\right\}
```

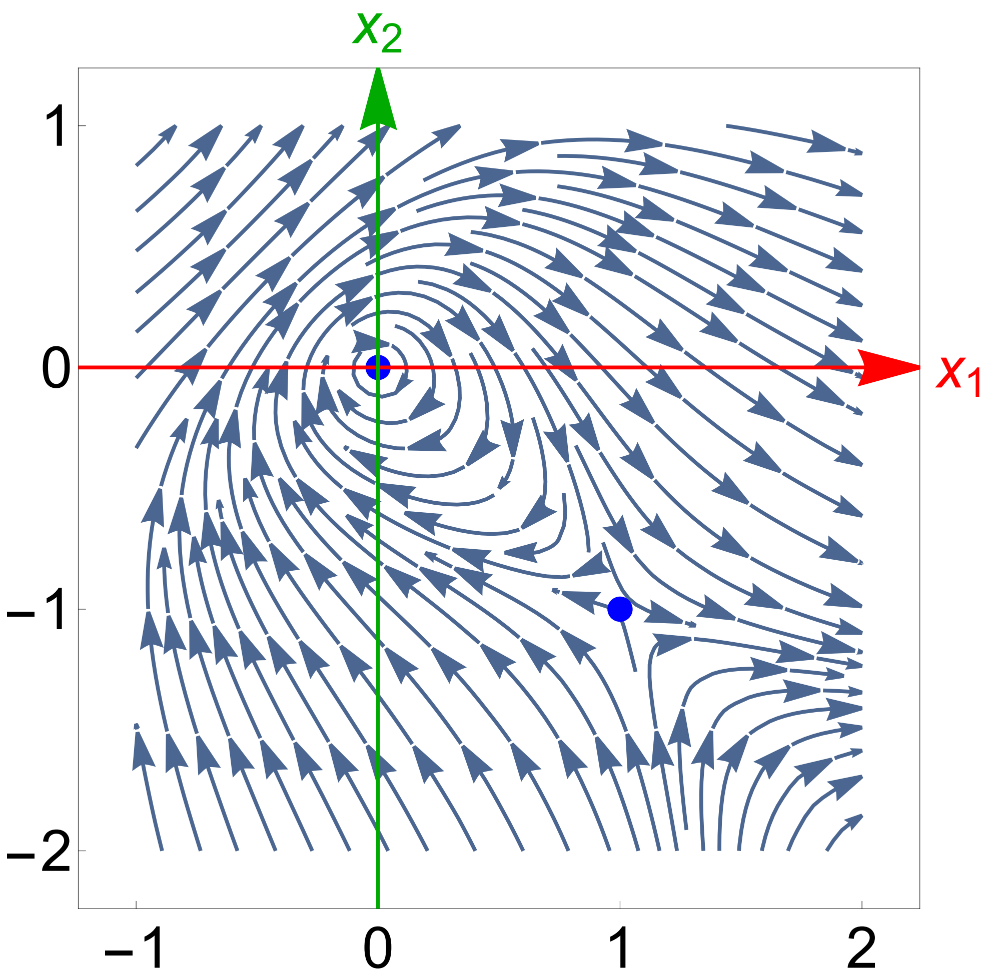
```
pts = Graphics[{Blue, PointSize[0.03], Point[{x, y} /. solpts]}];
```

```
vp = VectorPlot[Transpose[List @@ # & /@ eqs][[2]] /. {x[t] → x, y[t] → y},
  {x, -1, 2}, {y, -2, 1}, VectorScale → {0.03, Automatic, None}, Axes → True,
  GridLines → None, BaseStyle → FontSize → 20, VectorPoints → 30]
```



```
stream = StreamPlot[Transpose[List @@ # & /@ eqs][[2]] /. {x[t] → x, y[t] → y},
   {x, -1, 2}, {y, -2, 1}, Axes → True, GridLines → None, BaseStyle → FontSize → 30,
   FrameTicks → {{{-2, -1, 0, 1}, None}, {{-1, 0, 1, 2}, None}}, AxesStyle →
    {{Red, Thick, Arrowheads[0.07]}, {Green // Darker, Thick, Arrowheads[0.07]}},
   AxesLabel → {"x1", "x2"}, StreamStyle -> Thick, StreamScale -> Large];
```

```
Show[stream, pts]
```



Bilder zu Lösungskurven, diesmal instabiler und stabiler Knoten

allgemeine Suche

```
f[x_, y_] := a x² + b y - 2 x; g[x_, y_] := c y² - 0 x² + y;
```

$$J = \begin{pmatrix} \partial_x f[x, y] & \partial_y f[x, y] \\ \partial_x g[x, y] & \partial_y g[x, y] \end{pmatrix}$$

```
{{-2 + 2 a x, b}, {0, 1 + 2 c y}}
```

```
eig = Eigenvalues[J]
```

```
{-2 + 2 a x, 1 + 2 c y}
```

```
fix = Solve[{f[x, y] == 0, g[x, y] == 0}, {x, y}]
```

$$\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{2}{a}, y \to 0\right\}, \left\{x \to \frac{c - \sqrt{a b c + c^2}}{a c}, y \to -\frac{1}{c}\right\}, \left\{x \to \frac{c + \sqrt{a b c + c^2}}{a c}, y \to -\frac{1}{c}\right\}\right\}$$

```
fixeig = eig /. fix // FullSimplify
```

$\left\{\{-2, 1\}, \{2, 1\}, \left\{-\frac{2\sqrt{c\,(a\,b+c)}}{c}, -1\right\}, \left\{\frac{2\sqrt{c\,(a\,b+c)}}{c}, -1\right\}\right\}$

```
fixeig /. {a → 1, b → 5, c → 4}
```

$\{\{-2, 1\}, \{2, 1\}, \{-3, -1\}, \{3, -1\}\}$

nun eingesetzt

```
f[x_, y_] := 4 x^2 + 5 y - 2 x; g[x_, y_] := y^2 + y;
g[x, y]
```

$y + y^2$

$J = \begin{pmatrix} \partial_x f[x, y] & \partial_y f[x, y] \\ \partial_x g[x, y] & \partial_y g[x, y] \end{pmatrix}$

$\{\{-2 + 8\,x, 5\}, \{0, 1 + 2\,y\}\}$

```
eig = Eigenvalues[J]
```

$\{-2 + 8\,x, 1 + 2\,y\}$

```
fix = Solve[{f[x, y] == 0, g[x, y] == 0}, {x, y}]
```

$\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{1}{2}, y \to 0\right\}, \left\{x \to \frac{1}{4}\left(1 - \sqrt{21}\right), y \to -1\right\}, \left\{x \to \frac{1}{4}\left(1 + \sqrt{21}\right), y \to -1\right\}\right\}$

```
fixeig = eig /. fix // PowerExpand // Simplify
```

$\left\{\{-2, 1\}, \{2, 1\}, \left\{-2\sqrt{21}, -1\right\}, \left\{2\sqrt{21}, -1\right\}\right\}$

Bilder dazu erzeugen

```
eqs = {x'[t] == 4 x[t]^2 + 5 y[t] - 2 x[t], y'[t] == y[t] + y[t]^2};
solpts = Solve[eqs /. {x'[t] → 0, y'[t] → 0, x[t] → x, y[t] → y}]
```

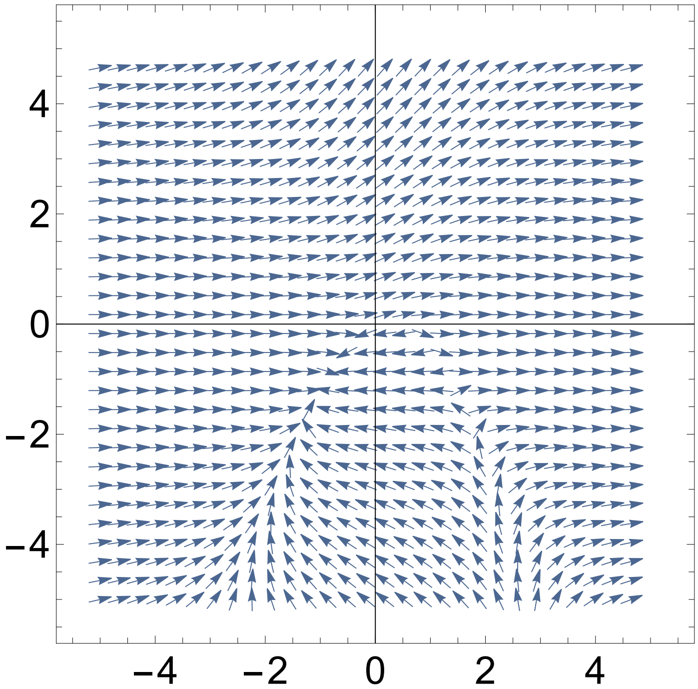
$\left\{\{x \to 0, y \to 0\}, \left\{x \to \frac{1}{2}, y \to 0\right\}, \left\{x \to \frac{1}{4}\left(1 - \sqrt{21}\right), y \to -1\right\}, \left\{x \to \frac{1}{4}\left(1 + \sqrt{21}\right), y \to -1\right\}\right\}$

```
eqs /. {x'[t] → x₁', y'[t] → x₂', x[t] → x₁, y[t] → x₂} // TeXForm
```

```
\left\{x_1'=4 x_1^2-2 x_1+5
   x_2,x_2'=x_2^2+x_2\right\}
```

```
pts = Graphics[{Blue, PointSize[0.03], Point[{x, y} /. solpts]}];
```

```
vp = VectorPlot[Transpose[List @@ # & /@ eqs][[2]] /. {x[t] → x, y[t] → y},
  {x, -5, 5}, {y, -5, 5}, VectorScale → {0.03, Automatic, None}, Axes → True,
  GridLines → None, BaseStyle → FontSize → 20, VectorPoints → 30]
```




```
stream = StreamPlot[Transpose[List @@ # & /@ eqs][[2]] /. {x[t] → x, y[t] → y},
   {x, -1.5, 2}, {y, -1.5, 0.5}, Axes → True, GridLines → None, BaseStyle → FontSize → 30,
   FrameTicks → {{{-1.5, -1, -0.5, 0, 0.5}, None}, {{-1, 0, 1, 2}, None}}, AxesStyle →
    {{Red, Thick, Arrowheads[0.07]}, {Green // Darker, Thick, Arrowheads[0.07]}},
   AxesLabel → {"x1", "x2"}, StreamStyle -> Thick,
   StreamScale -> Large, StreamPoints → Fine];
```

```
Show[stream, pts]
```