Mehrfachintegrale - Bereichsintegrale

Buch:Höhere Mathematik sehen und verstehen, Haftendorn, Riebesehl, Dammer, Springer Spektrum, Feb. 2021

Datei int3d-fl-unter-ober.nb zu Abschnitt 3.4.1 Seite 260, Abb. 3.27

Bilder zu Bereichsintegralen

Eine nette Funktion von 2 Veränderlichen

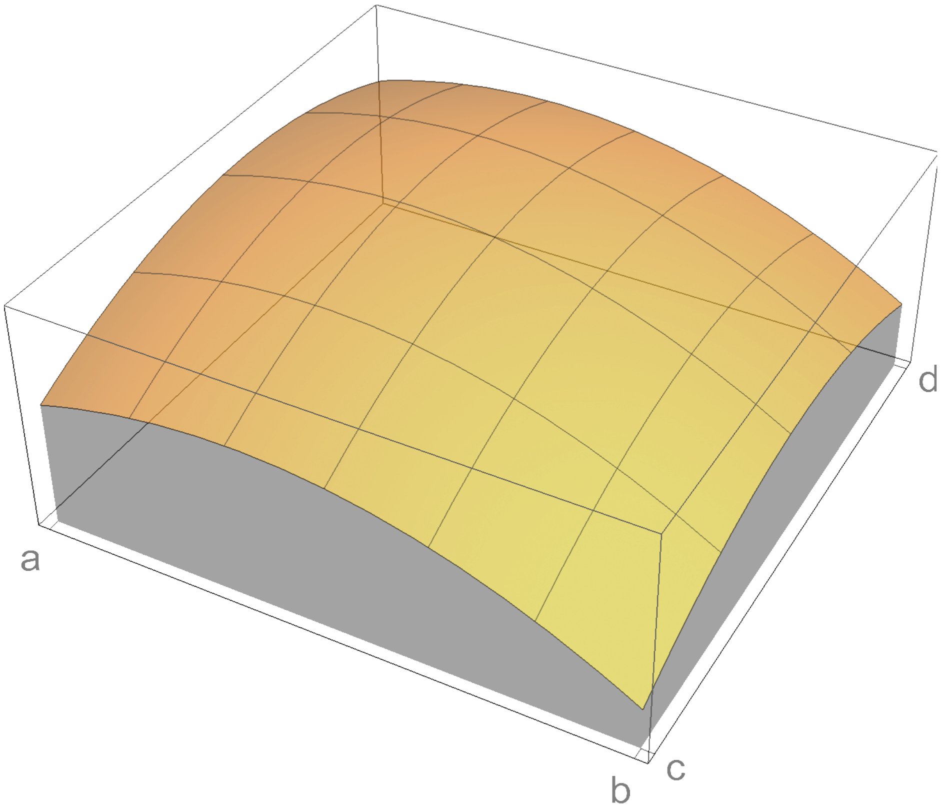
In[∘]:= $f[x\_, y\_] := 2 - x^2 - \frac{1}{2}\left(y - \frac{1}{8}\right)^2$

Bild dazu

```
In[∘]:= fplot[op_] := Plot3D[f[x, y], {x, -0.5, 1}, {y, -1, 1}, Mesh → {5, 3}, Filling → Axis,
          PlotRange → {0, Automatic}, BaseStyle → {FontSize → 20, Opacity[op]},
          Ticks → {{{-0.5, "a"}, {1, "b"}}, {{-1, "c"}, {1, "d"}}, None}]
```

```
In[∘]:= fplot[0.5] = Plot3D[f[x, y], {x, -0.5, 1}, {y, -1, 1}, Mesh → {5, 3}, Filling → Axis,
          PlotRange → {0, Automatic}, BaseStyle → {FontSize → 20, Opacity[0.5]},
          Ticks → {{{-0.5, "a"}, {1, "b"}}, {{-1, "c"}, {1, "d"}}, None}]
```

Out[∘]=



Die Punkte unter dem Gitter in der x-y-Ebene finden:

```
In[∘]:= pt = Table[{x, y}, {x, -0.5, 1, 0.25}, {y, -1, 1, 0.5}];
```

Die Quader über jedem Raster-Rechteck finden, die eine Untersumme bilden:

In[∘]:=
```
unter = Table[
   Cuboid[{pt[[i, j]], 0} // Flatten, {pt[[i + 1, j + 1]], Min[f[Sequence @@ pt[[i, j]]],
       f[Sequence @@ pt[[i + 1, j]]], f[Sequence @@ pt[[i, j + 1]]],
       f[Sequence @@ pt[[i + 1, j + 1]]]]} // Flatten], {i, 1, 6}, {j, 1, 4}];
```

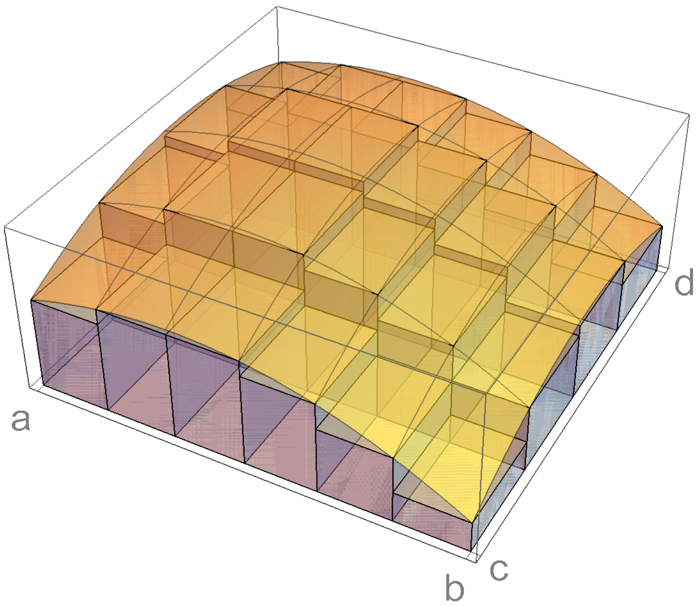
Die Quader über jedem Raster-Rechteck finden, die eine Obersumme bilden:

In[∘]:=
```
ober = Table[
   Cuboid[{pt[[i, j]], 0} // Flatten, {pt[[i + 1, j + 1]], Max[f[Sequence @@ pt[[i, j]]],
       f[Sequence @@ pt[[i + 1, j]]], f[Sequence @@ pt[[i, j + 1]]],
       f[Sequence @@ pt[[i + 1, j + 1]]]]} // Flatten], {i, 1, 6}, {j, 1, 4}];
```

Und beides, mit durchscheinender Fläche, zeigen:

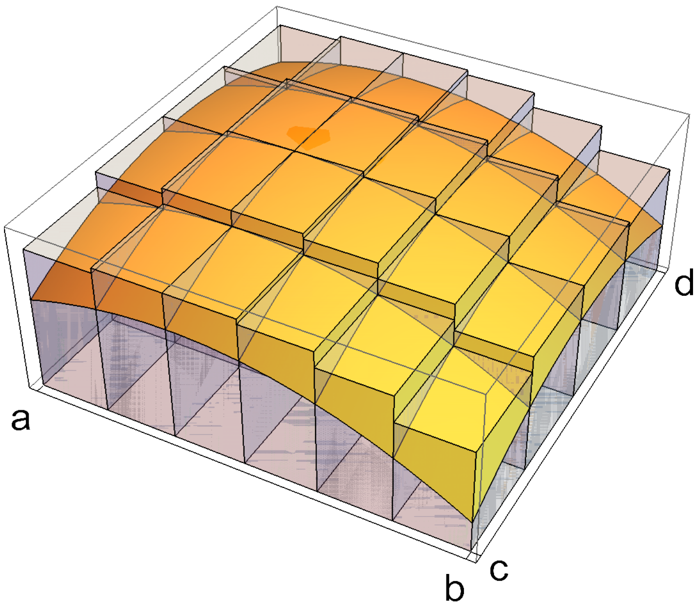
In[∘]:=
```
b1 = Show[fplot[0.5], Graphics3D[unter]]
```

Out[∘]=



In[∘]:=
```
b2 = Show[fplot[1], Graphics3D[{Opacity[0.3], ober}]]
```

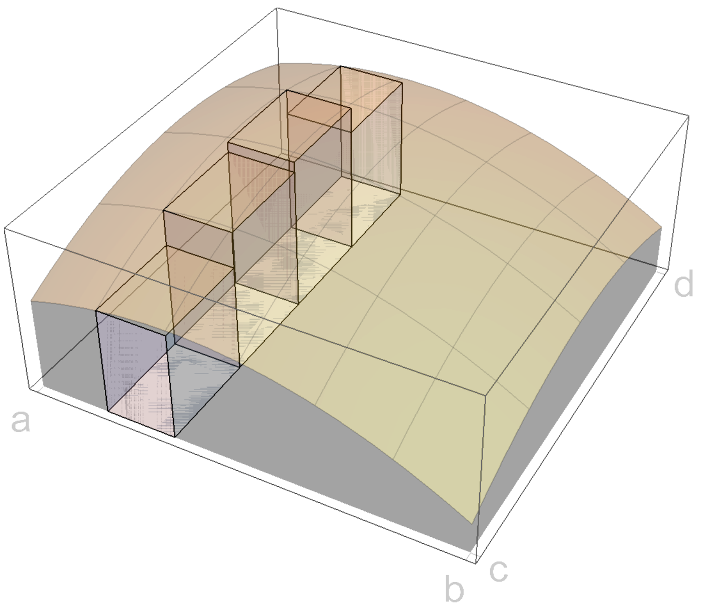
Out[∘]=



In[∘]:=
```
unter1 = Table[
   Cuboid[{pt[[i, j]], 0} // Flatten, {pt[[i + 1, j + 1]], Min[f[Sequence @@ pt[[i, j]]],
       f[Sequence @@ pt[[i + 1, j]]], f[Sequence @@ pt[[i, j + 1]]],
       f[Sequence @@ pt[[i + 1, j + 1]]]]} // Flatten], {i, 2, 2}, {j, 1, 4}];
```

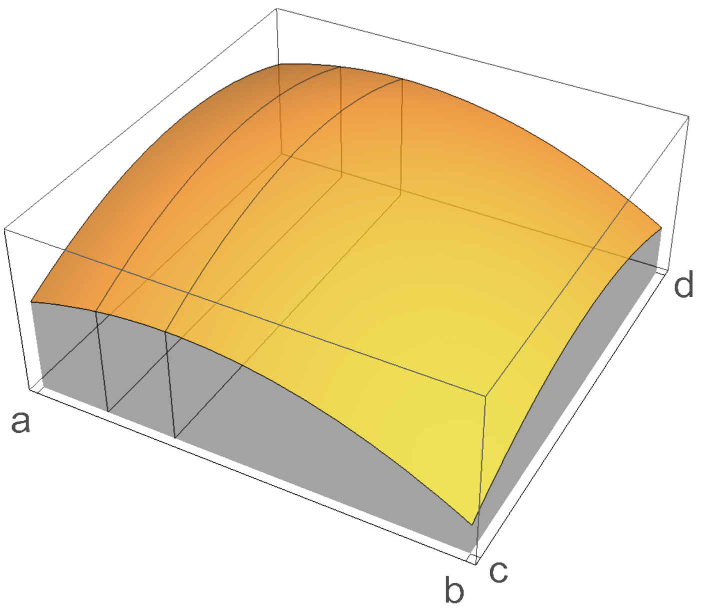
In[∘]:=
```
b3 = Show[fplot[0.2], Graphics3D[(*Opacity[0.1],*) unter1]]
```

Out[∘]=




```
b4 = Show[Plot3D[f[x, y], {x, -0.5, 1}, {y, -1, 1}, Mesh → {{-0.25, 0}, {-1, 1}},
   Filling → Axis, PlotRange → {0, Automatic}, BaseStyle → {FontSize → 20, Opacity[0.7]},
   Ticks → {{{-0.5, "a"}, {1, "b"}}, {{-1, "c"}, {1, "d"}}, None}], Graphics3D[
   {Thickness[0.01], Line[{{0, -1, f[0, -1]}, {0, -1, 0}, {0, 1, 0}, {0, 1, f[0, 1]}}],
    Line[{{-0.25, -1, f[-0.25, -1]}, {-0.25, -1, 0},
      {-0.25, 1, 0}, {-0.25, 1, f[-0.25, 1]}}]}]]
```

Out[∘]=



In[◦]:=
```
b5 = Show[Plot3D[f[x, y], {x, -0.5, 1}, {y, -1, 1}, Mesh → {{-0.25}, {-1, 1}},
   Filling → Axis, PlotRange → {0, Automatic}, BaseStyle → {FontSize → 20, Opacity[0.7]},
   Ticks → {{{-0.5, "a"}, {1, "b"}}, {{-1, "c"}, {1, "d"}}, None}],
  Graphics3D[{Line[{{-0.25, -1, f[-0.25, -1]}, {-0.25, -1, 0},
      {-0.25, 1, 0}, {-0.25, 1, f[-0.25, 1]}}]}]]
```

Out[◦]=