

## ■ Hinleitung zur Fouriertransformation

Buch: Höhere Mathematik sehen und verstehen, Haftdorn, Riebesehl, Dammer,  
Springer Spektrum, Feb. 2021

Datei [FourierSeriesToTransformForTriangle.nb](#) zu Abschnitt .9.6 Seite 121, Abb. 1.81



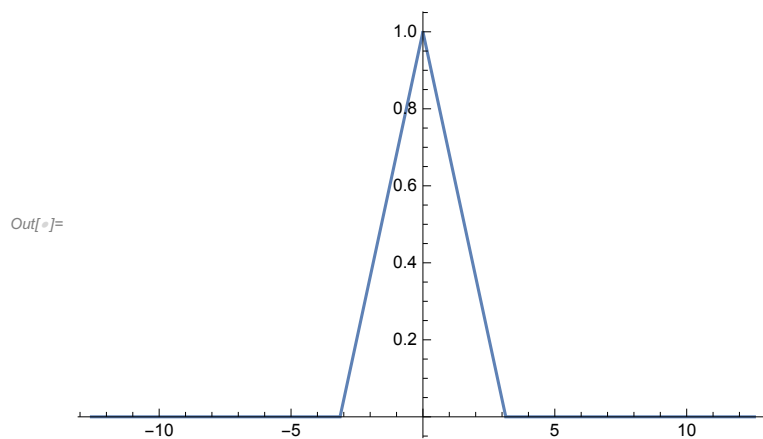
```
In[*]:= f[x_] := Piecewise[{{1, - $\frac{\pi}{2}$  ≤ x <  $\frac{\pi}{2}$ }, {0, True}}]
```

```
In[*]:= f[x_] := e-x2
```

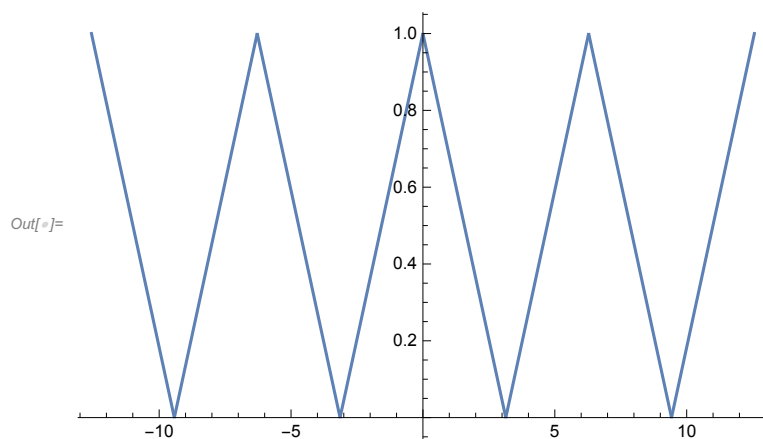
```
In[*]:= f[x_] :=  $\frac{1}{1+x^2}$ 
```

```
In[*]:= f[x_] := Piecewise[{{ $\frac{(x+\pi)}{\pi}$ , - $\pi$  ≤ x < 0}, { $\frac{(\pi-x)}{\pi}$ , 0 ≤ x <  $\pi$ }, {0, True}}]
```

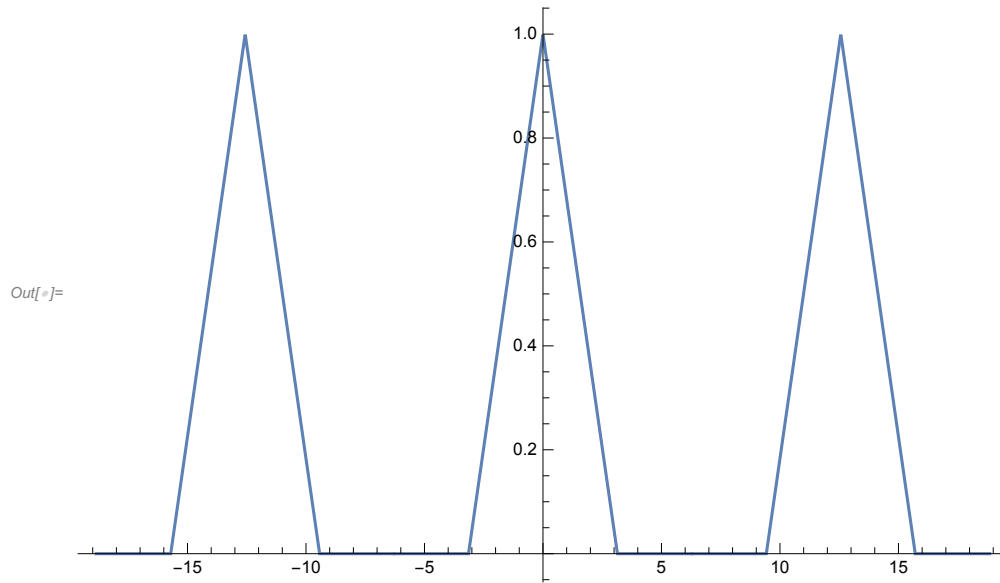
```
In[*]:= Plot[f[x], {x, -4  $\pi$ , 4  $\pi$ }, PlotRange → All]
```



```
In[*]:= Plot[f[Mod[x +  $\pi$ , 2  $\pi$ ] -  $\pi$ ], {x, -4  $\pi$ , 4  $\pi$ }, PlotRange → All]
```



```
In[ ]:= Plot[f[Mod[x + 2 π, 4 π] - 2 π], {x, -6 π, 6 π}, PlotRange -> All]
```



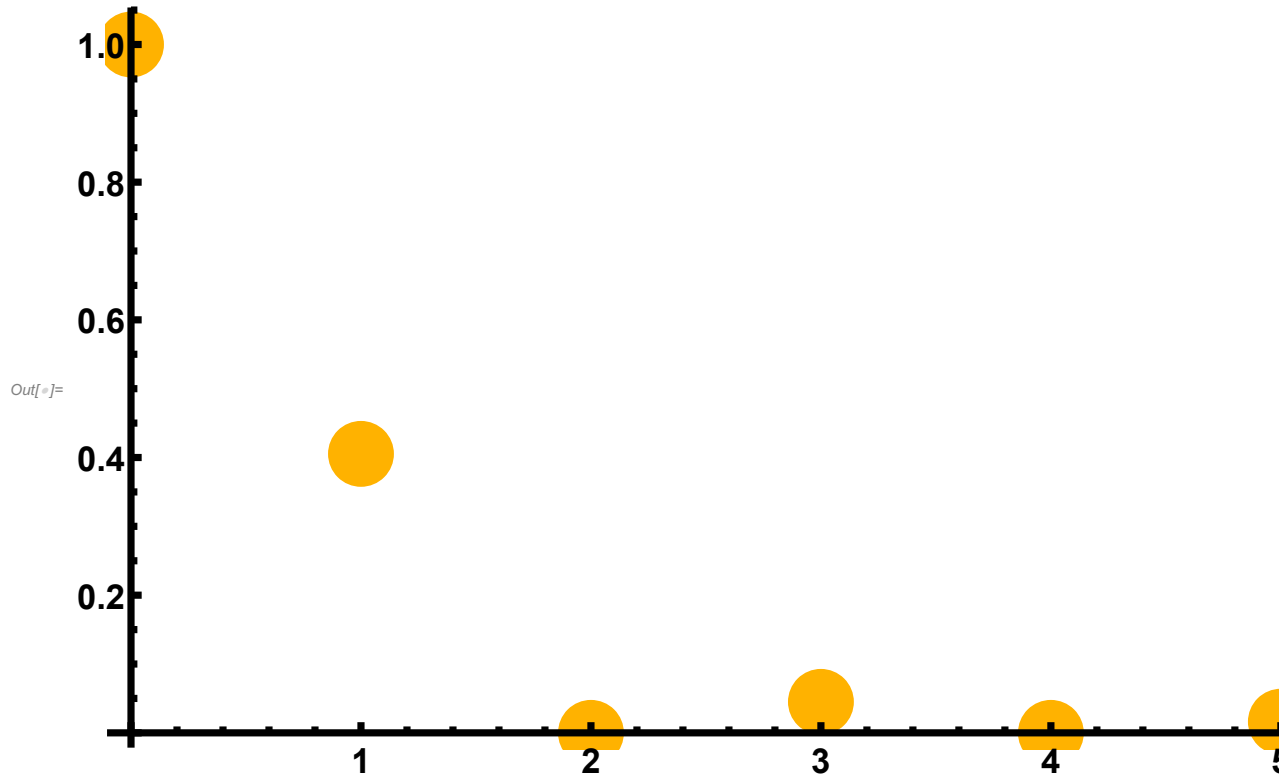
```
In[ ]:= s1 = Table[{n, 1/π Integrate[f[x] Cos[n x], {x, -π, π}], {n, 0, 5]}
```

```
Out[ ]:= {{0, 1}, {1, 4/π²}, {2, 0}, {3, 4/9π²}, {4, 0}, {5, 4/25π²}}
```

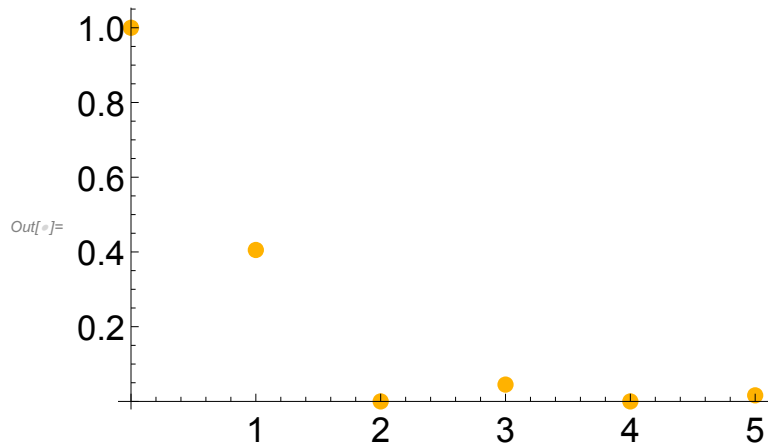
```
In[ ]:= Table[{n/2, 1/π Integrate[f[x] Cos[n x/2], {x, -2 π, 2 π}], {n, 0, 10]}
```

```
Out[ ]:= {{0, 1}, {1/2, 8/π²}, {1, 4/π²}, {3/2, 8/9π²}, {2, 0},
{5/2, 8/25π²}, {3, 4/9π²}, {7/2, 8/49π²}, {4, 0}, {9/2, 8/81π²}, {5, 4/25π²}}
```

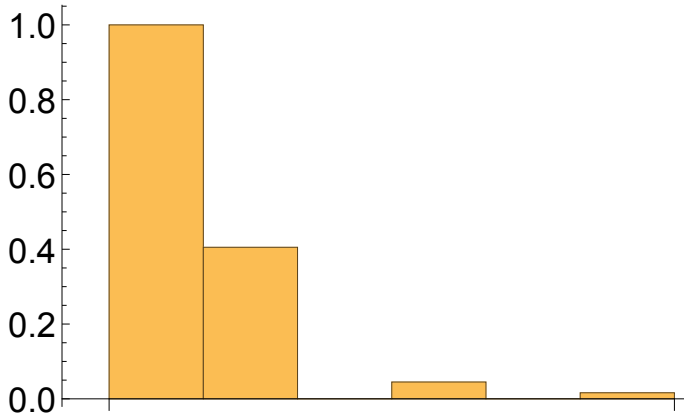
```
In[ ]:= p1 = ListPlot[s1, PlotStyle -> {PointSize[0.055`], RGBColor[1, 0.7, 0]},
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}},
  PlotRange -> All, AxesOrigin -> {0, 0}, BaseStyle -> {FontSize -> 18}]
```



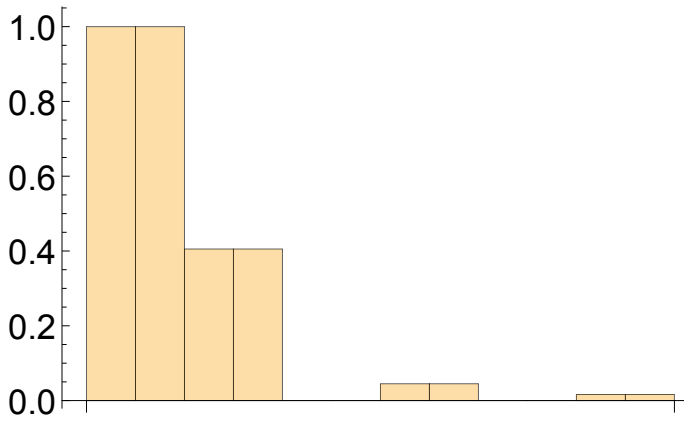
```
In[ ]:= p1s = ListPlot[{{#[[1]], 1#[[2]]} & /@ s1,
  PlotStyle -> {PointSize[0.025`], RGBColor[1, 0.7, 0]},
  PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



```
p1b = BarChart[Last /@ s1, PlotRange -> All, AxesOrigin -> {0, 0},
  BaseStyle -> {FontSize -> 18}, BarSpacing -> 0, ChartStyle -> Opacity[1]]
```

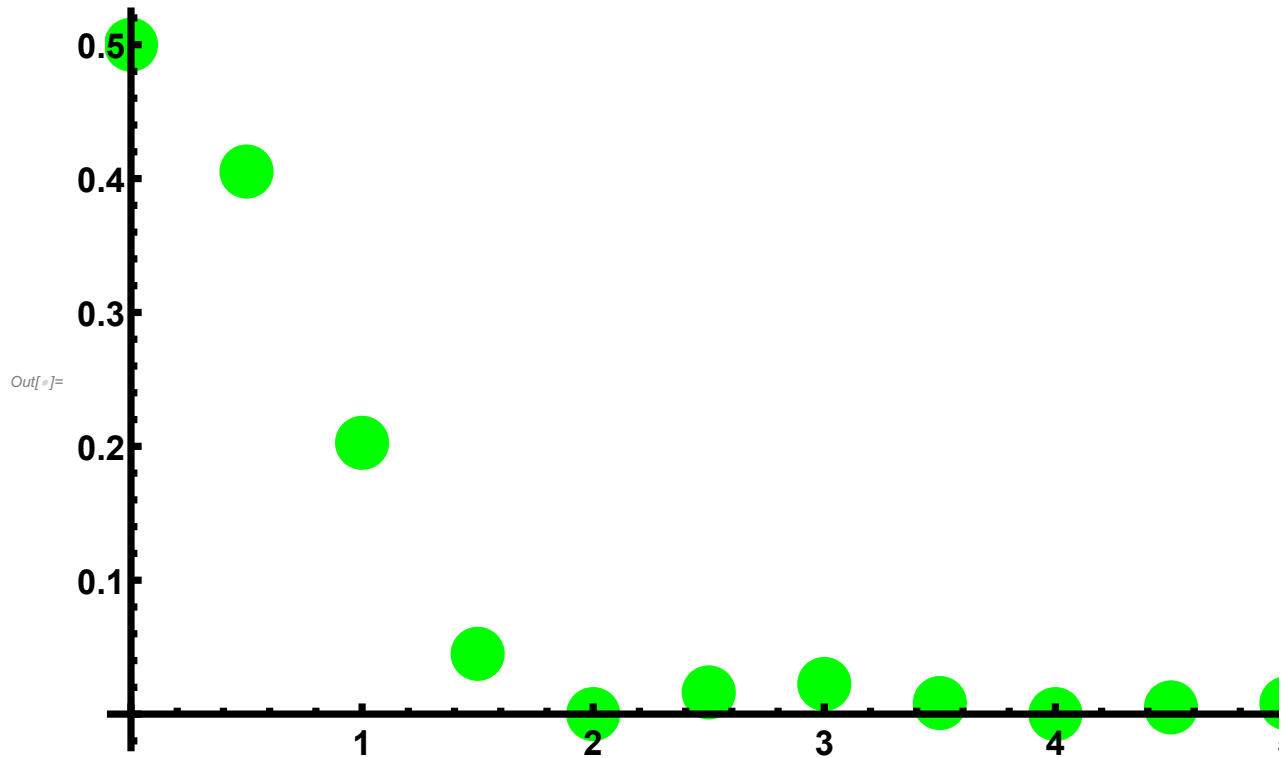


```
p1b = BarChart[{Last /@ s1, Last /@ s1} // Transpose // Flatten,
  PlotRange -> All, AxesOrigin -> {0, 0}, BaseStyle -> {FontSize -> 18},
  BarSpacing -> 0, ChartStyle -> Opacity[0.5]]
```

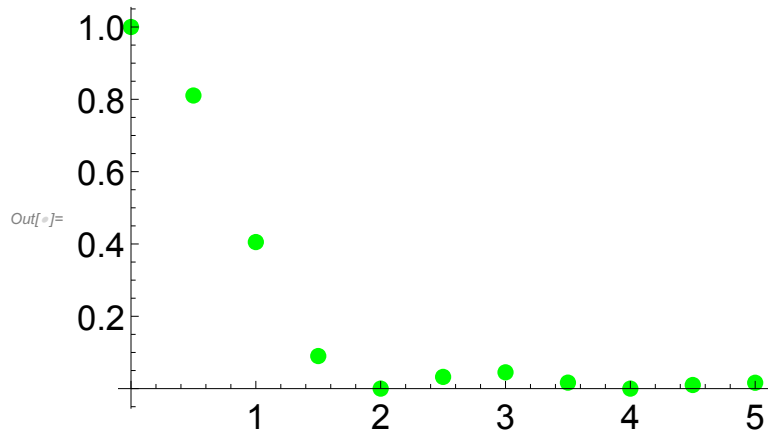


```
In[ ]:= s2 = Table[{n/2, 1. NIntegrate[f[x] Cos[n x/2], {x, -2 π, 2 π}]}, {n, 0, 10}];
```

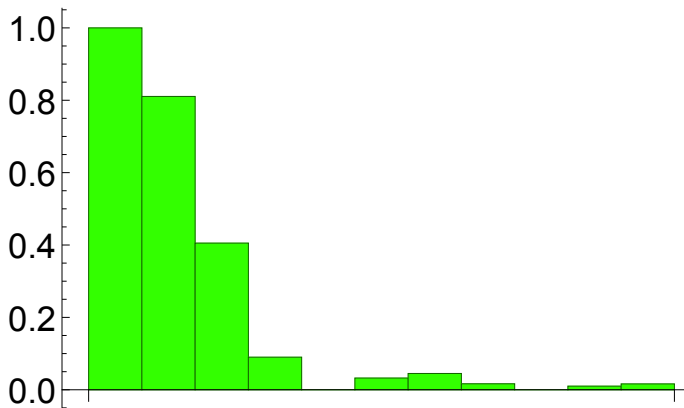
```
In[ ]:= p2 = ListPlot[s2, PlotStyle -> {PointSize[0.045`], RGBColor[0, 1, 0]},
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}},
  PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



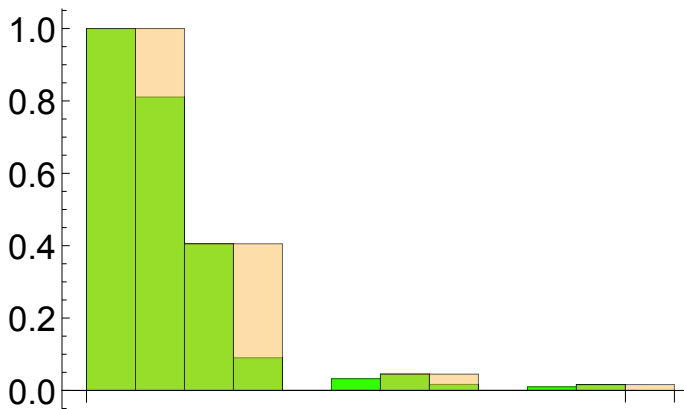
```
In[ ]:= p2s = ListPlot[{{#[[1]], 2#[[2]]} & /@ s2,
  PlotStyle -> {PointSize[0.025`], RGBColor[0, 1, 0]},
  PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



```
p2b = BarChart[2 Last /@ s2, PlotRange -> All, AxesOrigin -> {0, 0},
  BaseStyle -> {FontSize -> 18}, ChartStyle -> Hue[0.3], BarSpacing -> 0]
```

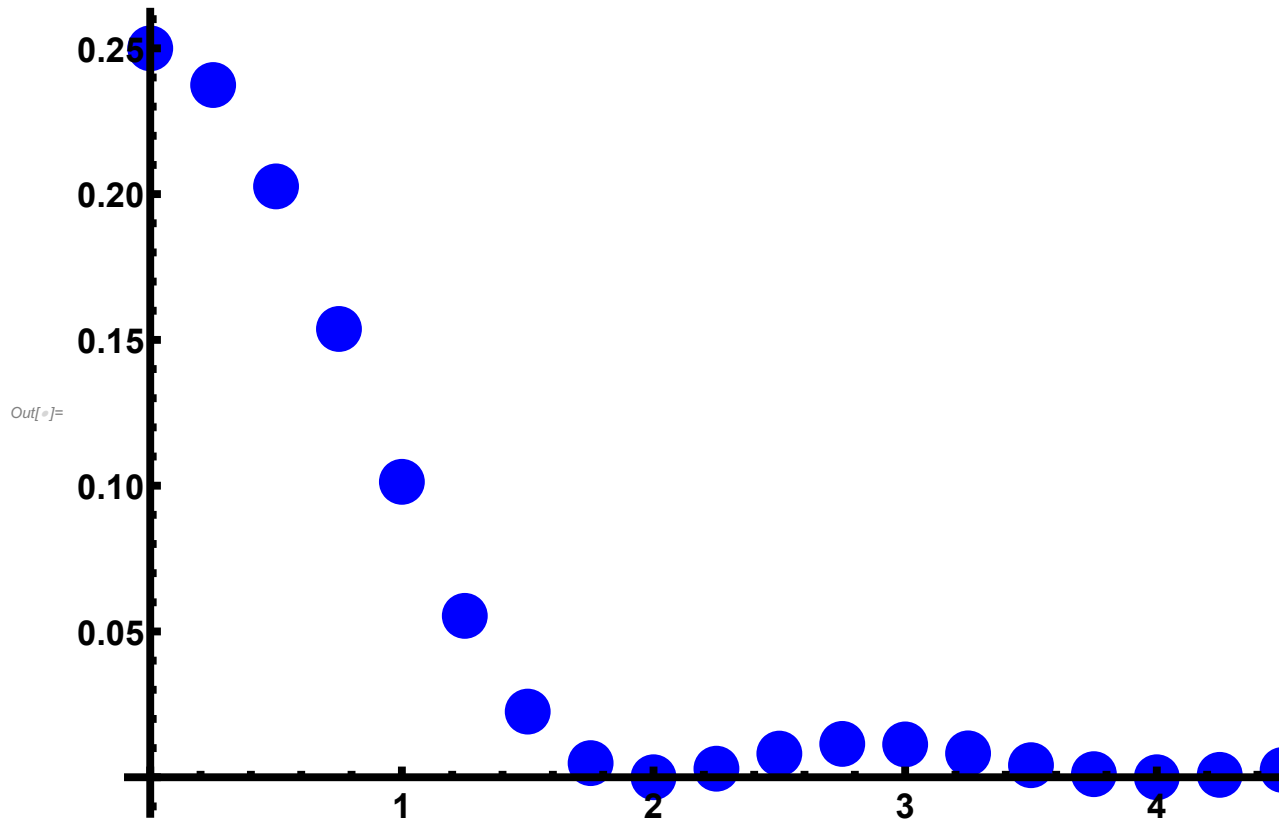


```
p21b = Show[p2b, p1b]
```

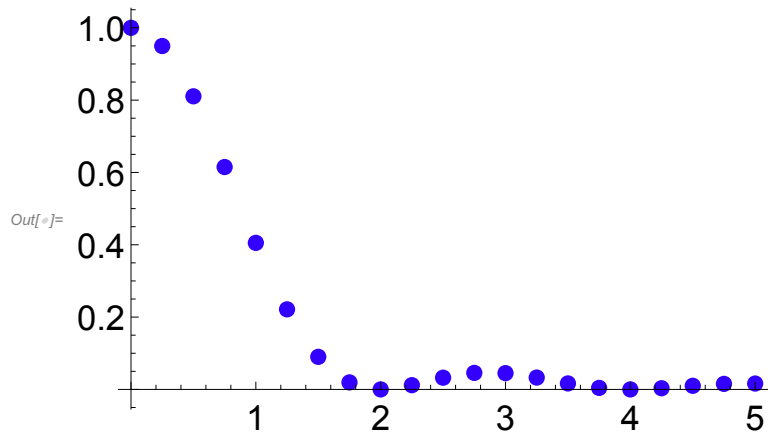


```
In[ ]:= s3 = Table[
  {n/4, 1/(4 π) NIntegrate[f[x] Cos[n x/4], {x, -4 π, 4 π}, MaxRecursion -> 10]}, {n, 0, 20}];
```

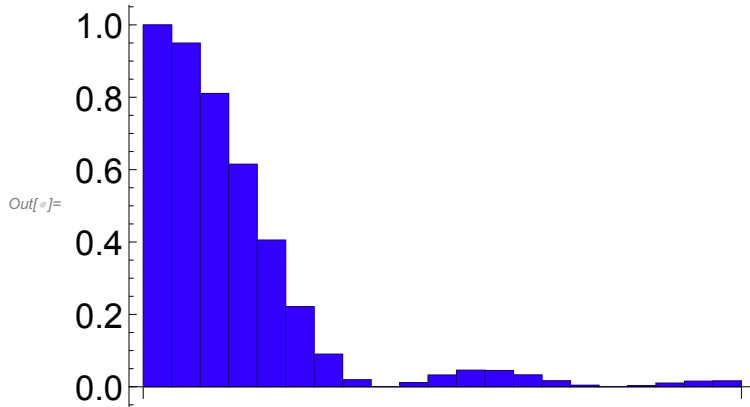
```
In[ ]:= p3 = ListPlot[s3, PlotStyle -> {PointSize[0.035`], RGBColor[0, 0, 1]},
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}},
  PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



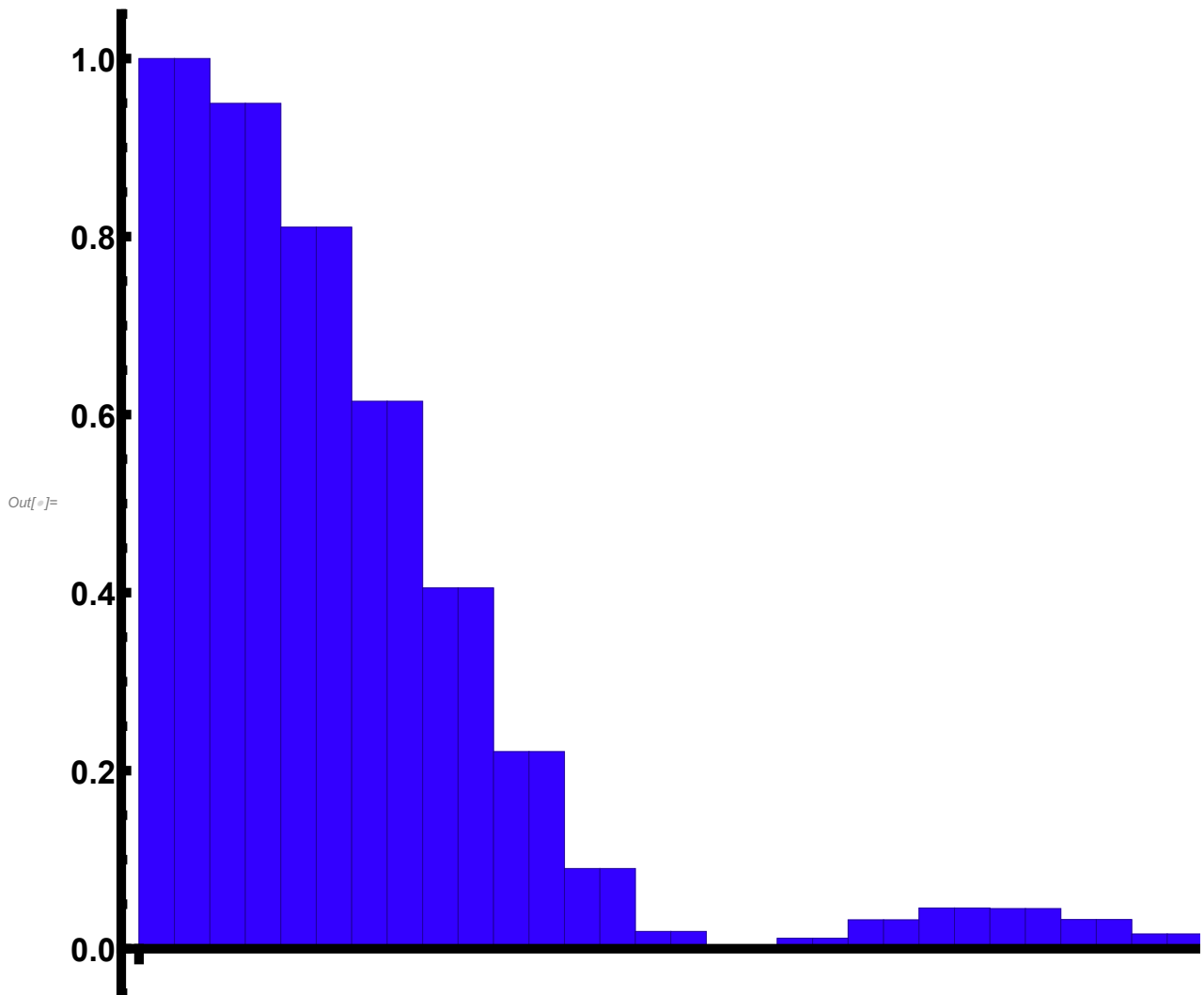
```
In[ ]:= p3s = ListPlot[{{#[[1]], 4#[[2]]} & /@ s3, PlotStyle -> {PointSize[0.025`], Hue[0.7`]},
  PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



```
In[ ]:= p3b = BarChart[4 Last /@ s3, PlotRange -> All, AxesOrigin -> {0, 0},
  BaseStyle -> {FontSize -> 18}, ChartStyle -> Hue[0.7], BarSpacing -> 0]
```



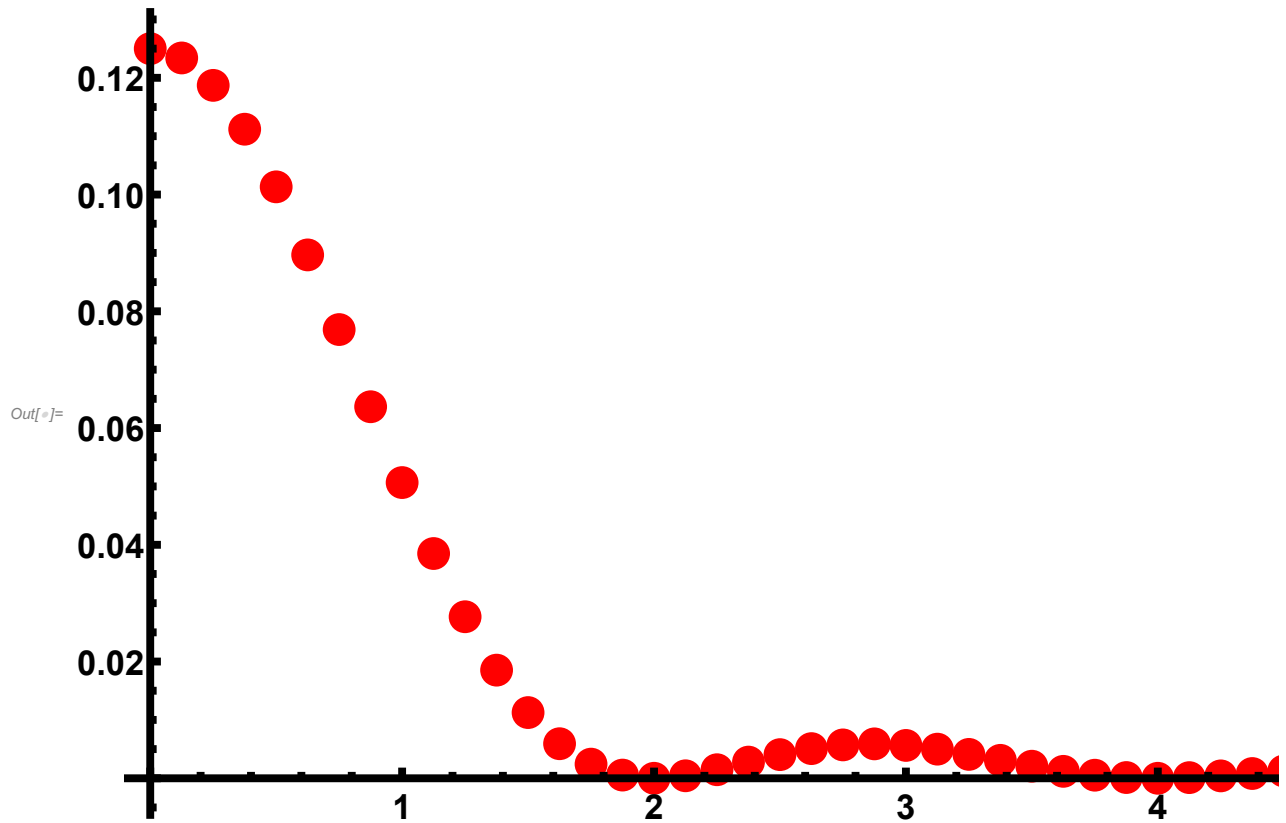
```
In[ ]:= p3c = BarChart[{4 Last /@ s3, 4 Last /@ s3} // Transpose // Flatten,
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}},
  PlotRange -> All, AxesOrigin -> {0, 0}, BaseStyle -> {FontSize -> 18},
  BarSpacing -> 0, ChartStyle -> {Hue[0.7]}]
```



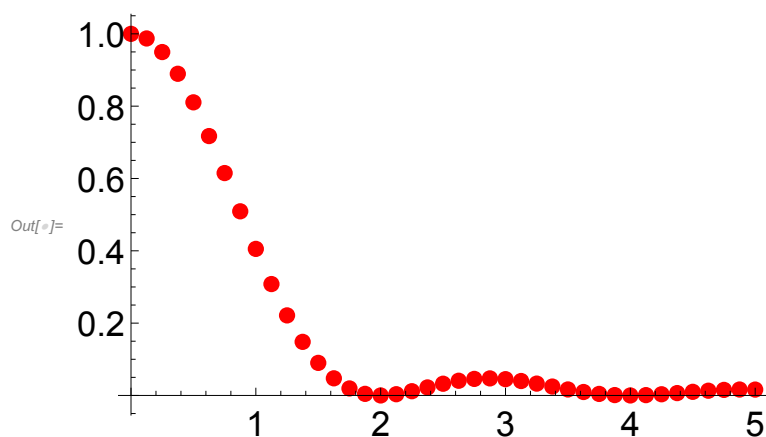


```
In[ ]:= s4 = Table[
  { $\frac{n}{8}$ ,  $\frac{1}{8\pi}$  NIntegrate[f[x] Cos[ $\frac{n x}{8}$ ], {x, -8  $\pi$ , 8  $\pi$ }, MaxRecursion -> 10]}, {n, 0, 40}];
```

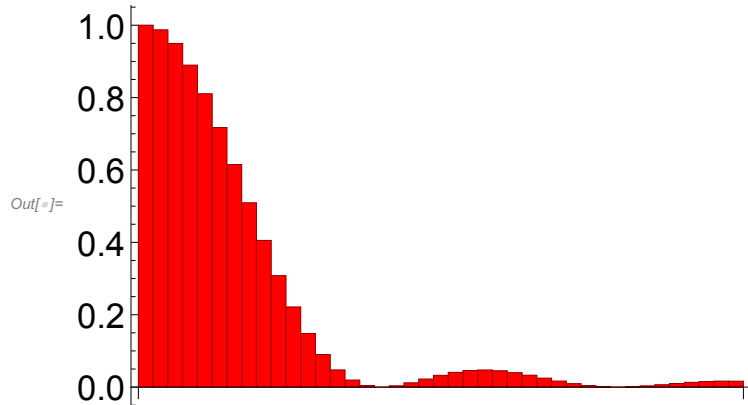
```
In[ ]:= p4 = ListPlot[s4, PlotStyle -> {PointSize[0.025`], RGBColor[1, 0, 0]},
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}},
  PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



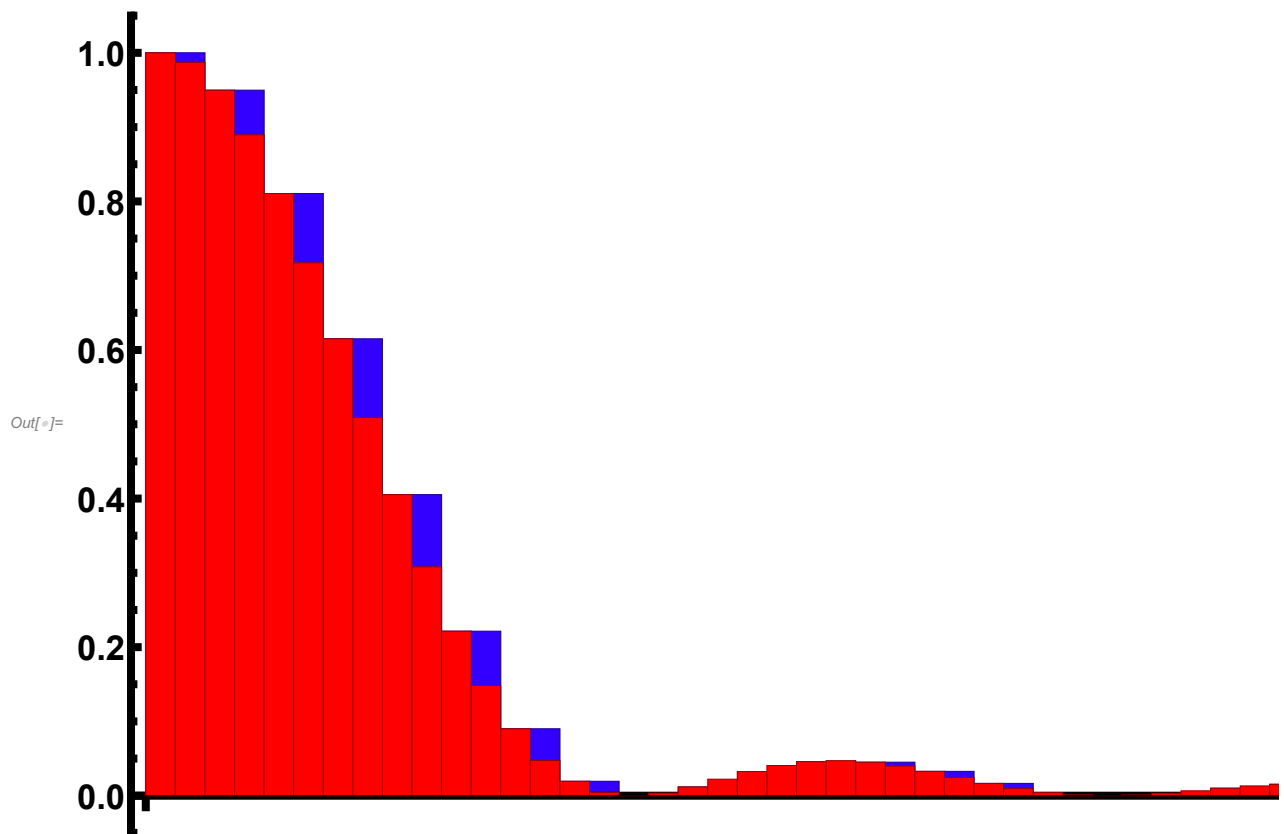
```
In[ ]:= p4s = ListPlot[{{#[[1]], 8#[[2]]} & /@ s4,
  PlotStyle -> {PointSize[0.025`], Hue[0]}, PlotRange -> All, BaseStyle -> {FontSize -> 18}]
```



```
In[ ]:= p4b = BarChart[8 Last /@ s4, PlotRange -> All,
  AxesOrigin -> {0, 0}, BaseStyle -> {FontSize -> 18},
  ChartStyle -> Hue[0], BarSpacing -> 0, ChartStyle -> Opacity[0.5]]
```



```
In[ ]:= p34 = Show[p3c, p4b]
```



```
In[ ]:= F[t_] := Simplify[ $\frac{1}{\pi} \int_{-\infty}^{\infty} f[x] \text{Cos}[t x] dx$ , t ∈ Reals] // Evaluate
```

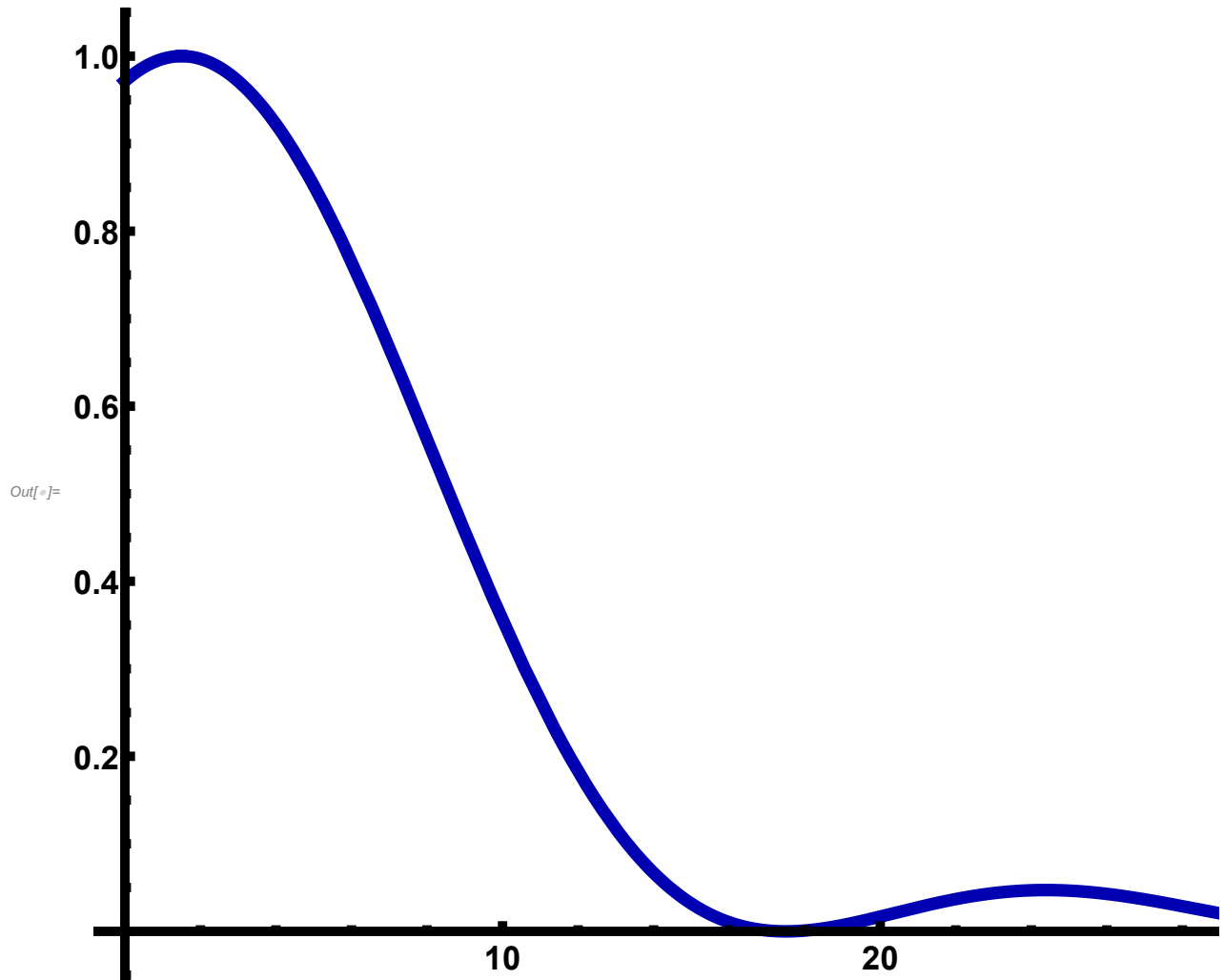
```
F[t] // TeXForm
```

```
 $\frac{2 - 2 \cos(\pi t)}{\pi^2 t^2}$ 
```

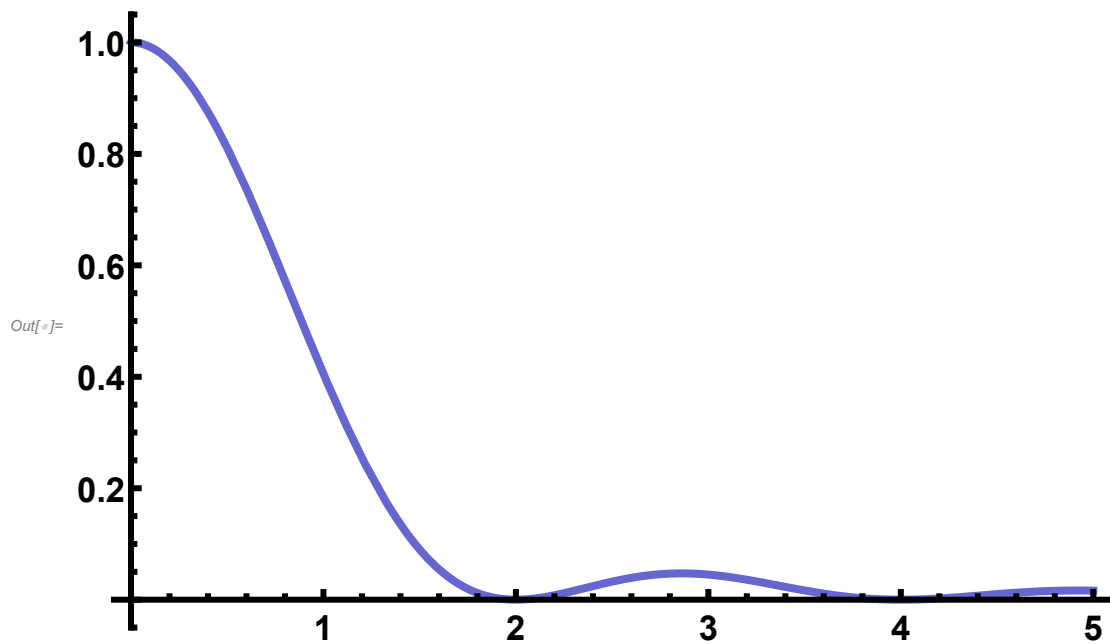
```
F[1]
```

```
 $\frac{4}{\pi^2}$ 
```

```
In[ ]:= Fps = Plot[F[t/8 - 1.5/8], {t, 0, 40},  
  PlotStyle -> {Thickness[0.008], RGBColor[0, 0, 0.7]}, AxesStyle ->  
  {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}}, BaseStyle -> {FontSize -> 18}]
```

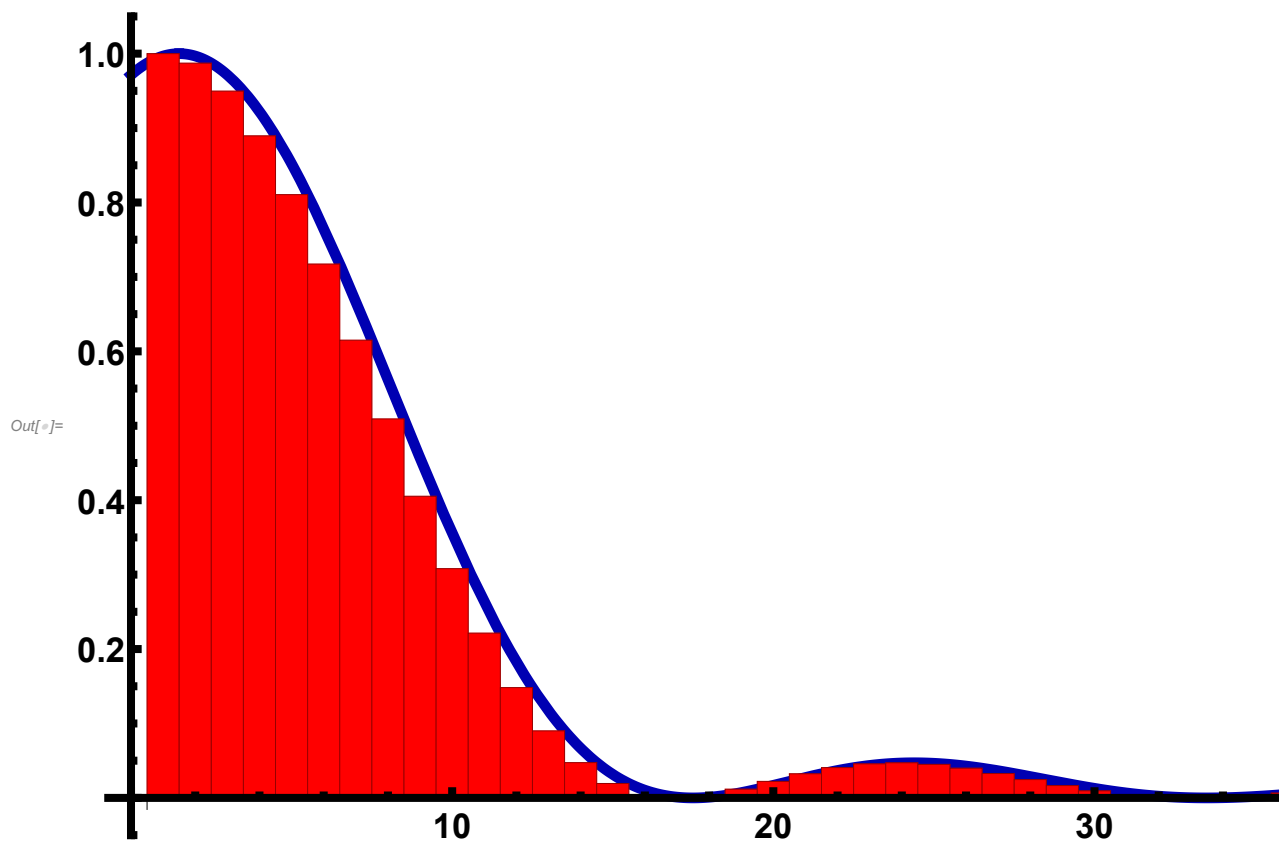


```
In[ ]:= Fp = Plot[F[t], {t, 0, 5},  
  PlotStyle -> {Thickness[0.008], RGBColor[0, 0, 0.7], Opacity[0.6]},  
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}},  
  BaseStyle -> {FontSize -> 18}]
```



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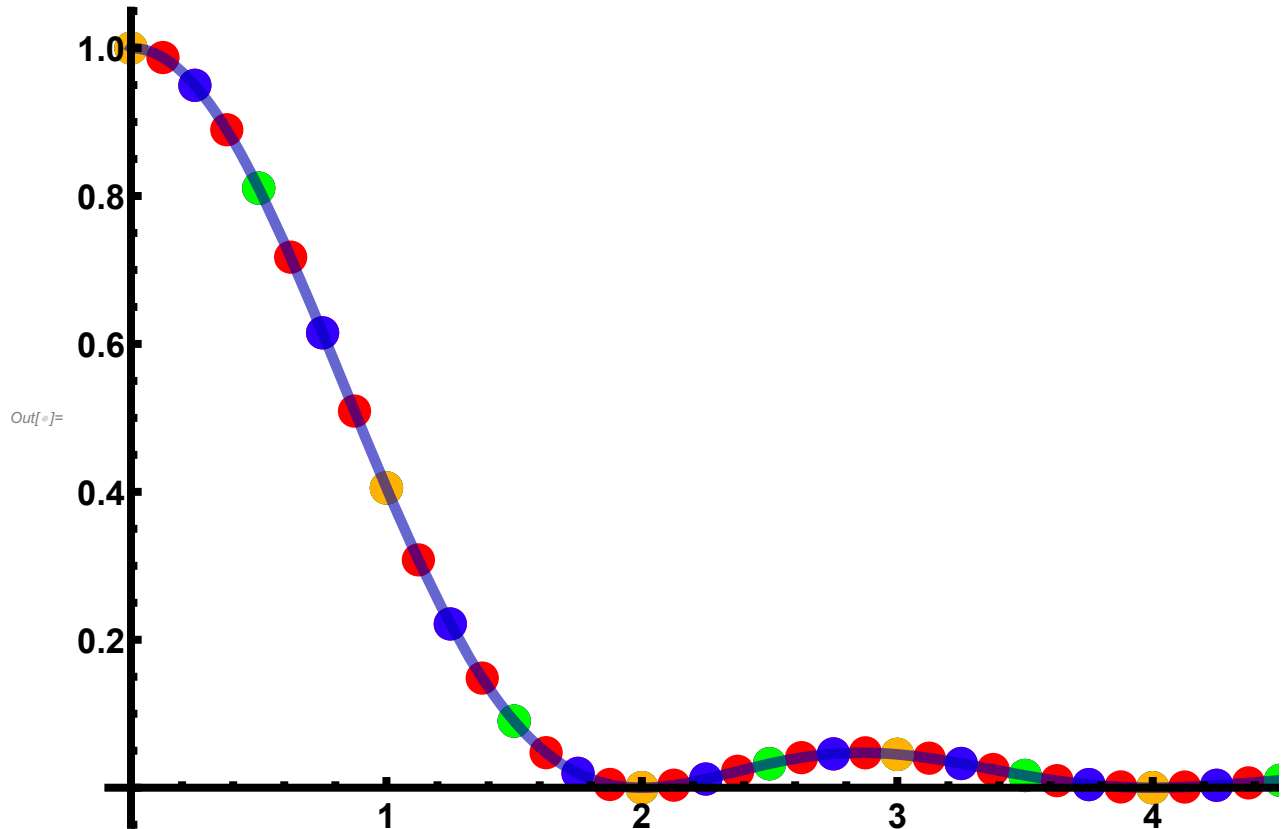
```
In[ ]:= Fp4b = Show[Fps, p4b]
```



$$\int_0^{\infty} e^{-\omega} \text{Cos}[\omega x] d\omega$$

$$\text{ConditionalExpression}\left[\frac{1}{1+x^2}, \text{Abs}[\text{Im}[x]] < 1\right]$$

```
In[ ]:= pall = Show[p4s, p3s, p2s, p1s, Fp,
  AxesStyle -> {{Thickness[0.006], Bold}, {Thickness[0.006], Bold}}, PlotRange -> All]
```



```
FourierCosTransform[f[x], x, t]
```

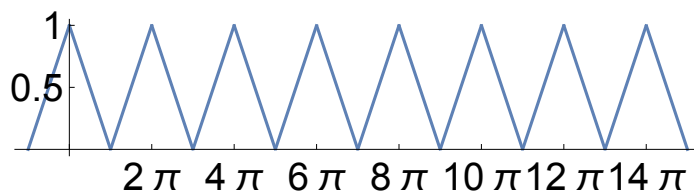
$$\frac{2 - 2 \text{Cos}[\pi t]}{\sqrt{2} \pi^{3/2} t^2}$$

```
FourierTransform[f[x], x, t]
```

$$\frac{e^{-i \pi t} (-1 + e^{i \pi t})^2}{\sqrt{2} \pi^{3/2} t^2}$$

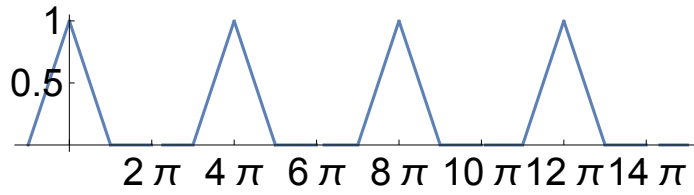
```
f1[x_] := f[Mod[x + \pi, 2 \pi] - \pi]
```

```
pf1 = Plot[f1[x], {x, -\pi, 15 \pi}, AspectRatio -> 0.2,
  Ticks -> {Table[2 i \pi, {i, 0, 7}], {0.5, 1}}, BaseStyle -> FontSize -> 20]
```



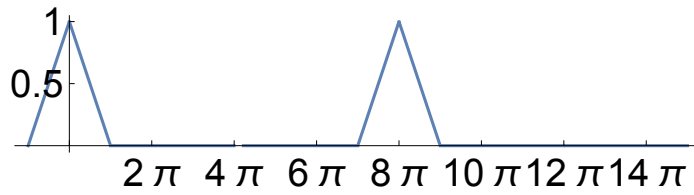
```
f2[x_] := f[Mod[x + 2 \pi, 4 \pi] - 2 \pi]
```

```
pf2 = Plot[f2[x], {x, -π, 15 π}, AspectRatio → 0.2,
  Ticks → {Table[2 i π, {i, 0, 7}], {0.5, 1}}, BaseStyle → FontSize → 20]
```



```
f3[x_] := f[Mod[x + 4 π, 8 π] - 4 π]
```

```
pf3 = Plot[f3[x], {x, -π, 15 π}, AspectRatio → 0.2,
  Ticks → {Table[2 i π, {i, 0, 7}], {0.5, 1}}, BaseStyle → FontSize → 20]
```



```
f4[x_] := f[Mod[x + 8 π, 16 π] - 8 π]
```

```
pf4 = Plot[f4[x], {x, -π, 15 π}, AspectRatio → 0.2,
  Ticks → {Table[2 i π, {i, 0, 7}], {0.5, 1}}, BaseStyle → FontSize → 20]
```

